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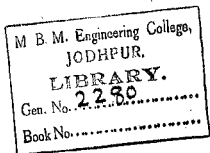
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# NOMOGRAPHY

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## PREFACE

In science and technology, nomograms are recognized for ease of operation and for the time saved in the repeated solution of mathematical formulas. The type of nomogram known as the alignment chart has won much favor during the past two decades. Engineers and scientists should be trained sufficiently to understand the mathematical theory and design of nomograms. The material presented in this book is intended to provide a good working knowledge of the fundamental principles.

Emphasis is placed upon the "geometric method" employed in the development of the theory for the design of alignment charts involving equations of three or more variables. Simple equations are solved by alignment charts consisting of straight-line scales, whereas more involved equations may necessitate the use of "grids," curved scales, and combinations of Cartesian co-ordinate charts with alignment charts.

Once the theory is well understood, practical short-cuts are presented to reduce the time required to design a chart. Examples are given in the chapter "Practical Short-Cuts."

The chapter on the use of determinants serves as an introduction to the "method of determinants" which has been well developed by such writers as d'Ocagne, Serreau, Hewes and Seward, and Allcock and Jones.

The appendix contains practical examples of alignment charts used in the fields of engineering, production, and statistics.

This book is an outgrowth of nomography courses offered for many years at both the University of Minnesota and the University of California. At present, one-semester, two-unit, elective courses are open for seniors in engineering, physics, chemistry, business administration, and education. Scientists and practicing engineers will find no difficulty in handling the material in this book, if they have not forgotten the elements of algebra and plane geometry and the use of logarithms.

For those who may develop a keen desire to pursue further study in the fascinating field of Nomography the selected bibliography will provide adequate reference material.

The author is indebted to both Emeritus Professor W. H. Kirchner, formerly head of the Department of Drawing and Descriptive Geometry, and Emeritus Professor W. E. Brooke, formerly head of the Department of Mathematics and Mechanics, at the University of Minnesota, for their guidance and encouragement during the early years of training and experience. Acknowledgments are due undergraduate and graduate students of the author's classes in Nomography, especially Russell M. Carlson, of the Engineering Department, Chance Vought Aircraft Division of United Aircraft Corporation, and colleagues of the Department of Engineering, University of California, for many valuable suggestions. The writer is grateful to Professors B. F. Raber and F. W. Hutchinson of the University of California, authors of "Refrigeration and Air Conditioning Engineering"; the Crane Company; Consolidated Vultee Aircraft Corporation; Crohlt, Inc.; Product Engineering; and Federal Telephone and Radio Corporation for permission to reproduce certain nomograms.

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August, 1947*

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# Chapter One

## INTRODUCTION

Pictorial representation has proved to be an effective method in conveying technical and semi-technical information to business and professional persons, to production personnel, to engineers, and to scientists.

Our daily newspapers, business magazines, company publications, technical papers, etc., invariably present graphs and charts which quickly and painlessly disclose information that would otherwise require lengthy descriptions if presented by the printed word. Figures 1, 2, and 3 are typical of such charts.

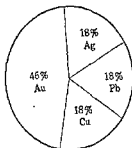


FIG. 1. Pie Chart.

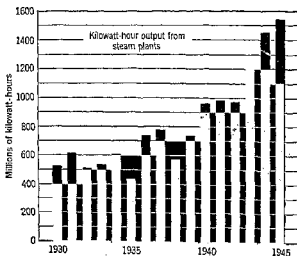


FIG. 2. Bar Chart.



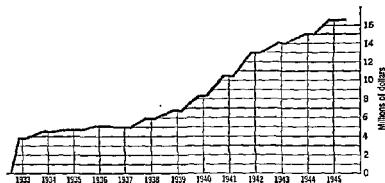
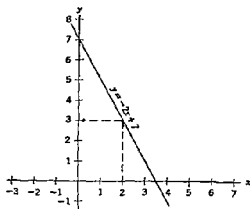


FIG. 3. Profile Chart.

Those engaged in technical work frequently employ the Cartesian co-ordinate system for the representation of the relationship between two or more variables. For example, the straight line  $y = -2x + 7$  is represented graphically by Figure 4. If  $x = 2$ , the value of  $y$  may be determined by following the arrows shown in the figure, which yields  $y = 3$ .

FIG. 4. Cartesian Co-ordinate Representation of the Equation,  $y = -2x + 7$ . Example: When  $x = 2$ ,  $y = 3$ .

Several equations of the form  $y = mx + b$  can, of course, be shown on one set of axes. For example, the equations

$$y = -2x + 1 \quad (1)$$

$$y = -2x + 2 \quad (2)$$

$$y = -2x + 3 \quad (3)$$

$$y = -2x + 5 \quad (4)$$

$$y = -2x - 4 \quad (5)$$

will appear as a family of parallel lines. (See Figure 5.)

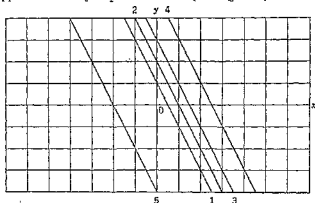


FIG. 5. Family of Parallel Lines. (1)  $y = -2x + 1$ . (2)  $y = -2x + 2$ . (3)  $y = -2x + 3$ . (4)  $y = -2x + 5$ . (5)  $y = -2x - 4$ .

An equation of the form  $f_1(u) + f_2(v) + f_3(w) = f_4(q)$  may be represented by a combination of Cartesian co-ordinate charts. For example, consider the relation  $u + v + w = q$ . Let

$$u + v = T \quad (1)$$

and 
$$T + w = q \quad (2)$$

Each of these equations is of the form  $f_1(u) + f_2(v) = f_3(w)$  and can be represented by a family of parallel lines. The first equation,  $u + v = T$ , is shown in Figure 6, and the second equation,  $T + w = q$ , is of similar form and is represented by Figure 7.

Obviously, not much is gained by solving the given equation,  $u + v + w = q$ , by parts as shown in Figures 6 and 7. A more effective arrangement would make it possible to use values of  $u$ ,  $v$ , and  $w$  directly without first determining the value of  $T$ . This is quite possible. First, let us analyze what happened in Figures 6 and 7. The equation  $u + v = T$ , written in the slope intercept form, would be  $v = -u + T$ ,

showing that for any value of  $T$ , we have a line with a negative slope of one and an intercept equal to  $T$ . Similarly, the equation  $T + w = q$  would be  $w = -T + q$ , showing that for any value of  $q$ , we have a line with a negative slope of one and an intercept equal to  $q$ .

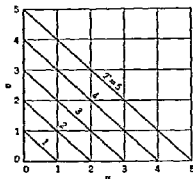


FIG. 6. Cartesian Co-ordinate Representation of the Equation,  $u + v = T$ .

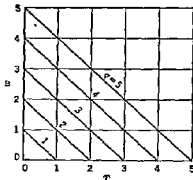


FIG. 7. Cartesian Co-ordinate Representation of the Equation,  $T + w = q$ .

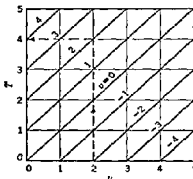


FIG. 8. Cartesian Co-ordinate Representation of the Equation,  $T = u + v$ . Example: When  $u = 2$  and  $v = 2$ ;  $T = 4$ .

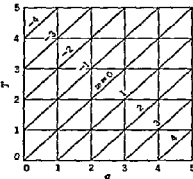


FIG. 9. Cartesian Co-ordinate Representation for the Equation,  $T = q - w$ .

Now, suppose we consider equation  $T = u + v$ , with  $T$  replacing  $y$  and  $u$  replacing  $x$  in the usual equation,  $y = mx + b$ . This will result in the graphic representation shown in Figure 8. Similarly, the equation  $T + w = q$  may be rewritten as  $T = q - w$ . The latter expression is shown graphically in Figure 9.

By combining the two charts as shown in Figure 10 it is now possible to eliminate the graduations of the  $T$  scale, and, in fact, the entire  $T$  scale, since it is common to both charts, and read  $u$ ,  $v$ ,  $w$ , and  $q$  directly. This method may be extended further for the solution of equations involving more than four variables.

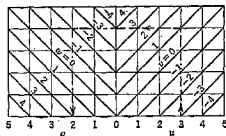


FIG. 10. Combination of Figures 8 and 9 for the Equation,  $u + v + w = q$ . Example: When  $u = 3$ ,  $v = 1$ , and  $w = -2$ ;  $q = 2$ . Follow the arrows shown in chart.

If the given equation is of the form  $uv = w$ , logarithmic scales may be employed, thus reducing the equation to  $\log u + \log v = \log w$ . Figure 11 shows the graphic representation of this equation.

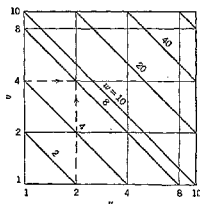


FIG. 11. Cartesian Co-ordinate Representation of the Equation,  $uv = w$ . Example: When  $u = 2$  and  $v = 4$ ;  $w = 8$ .

If the equation is of the form  $uvw = q$ , a nomogram can be designed as follows:

$$\text{Let} \quad uv = T \quad (1)$$

$$\text{and} \quad Tw = q \quad (2)$$

Equations 1 and 2 may be combined, graphically, as shown in Figure 12.

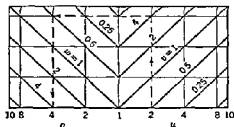


FIG. 12. Cartesian Co-ordinate Representation of the Equation,  $uvw = q$  Example: When  $u = 2$ ,  $v = 4$ , and  $w = 0.5$ ;  $q = 4$ .

Logarithmic scales may be eliminated by treating the equation  $uv = w$  in the following manner. Let  $y = w$  and  $x = u$ , from which  $y = vx$ . This equation represents a pencil of lines passing through the origin with slopes equal to values of  $v$  (Figure 13).

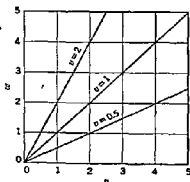


FIG. 13. Cartesian Co-ordinate Representation of the Equation,  $uv = w$ .

Thus an equation of the form  $uvw = q$  may be solved graphically by employing the methods set forth in previous examples.

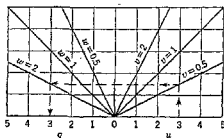


FIG. 14. Cartesian Co-ordinate Representation of the Equation,  $uvw = q$ . Example: When  $u = 3$ ,  $v = 0.5$ , and  $w = 2$ ;  $q = 3$ .

An example of the use of a combination of rectangular Cartesian co-ordinate charts is shown in Figure 15—"Graph to Determine Engine Rpm."

Another example is presented in Figure 16—"Graphical Evaluation of Heat Storage and Transfer Characteristics,  $Q'$  and  $q'$ ."

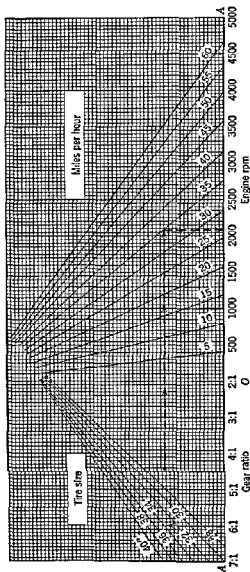


FIG. 15. Graph to Determine Engine Rpm (Courtesy of Federal-Mogul Corporation, Detroit, Mich.)

$$\text{Based on formula: Engine rpm (revolutions per minute)} = \frac{336 \times R \times S}{D}$$

where  $R$  = gear ratio,  $S$  = road speed of vehicle,  $D$  = tire size, nominal outside diameter

*Example* In a motor vehicle traveling at 35 mph, having an axle ratio of 3 to 1 and 34-in. tires, what is the engine speed in revolutions per minute?

*Solution* (see chart) Locate point of 3 to 1 axle ratio on base line  $A-A$  and trace upwards following the arrows to the diagonal line representing the 34-in. tire size. From the intersection of these two lines, draw a line straight across (following the arrow) to the diagonal line representing 35 mph. From that intersection, draw a straight line downwards to the base line  $A-A$ . According to the calibration on base line  $A-A$ , the engine speed is 2675 rpm.

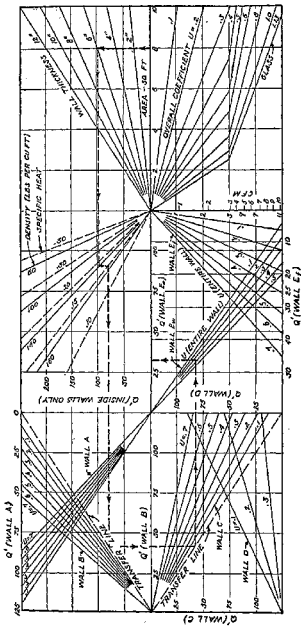


Fig. 16. Graphical Evaluation of Heat Storage and Transfer Characteristics,  $Q'$  and  $q'$ . (From B. F. Haber and F. W. Hutchinson's *Refrigeration and Air Conditioning Engineering*, John Wiley and Sons, New York, 1945. Reprinted with permission.)

Wall A. Plain; unfinished surface inside and outside. Wall B. Plain outside;  $\frac{1}{2}$ -in. plaster on inside, (turreted) inside. Wall D. Plain outside;  $\frac{1}{2}$ -in. plaster (turreted) inside, with  $\frac{1}{2}$ -in. insulation in furrow space. Wall E<sub>10</sub>. Same as E<sub>9</sub> but with 4-in. cut-stone veneer. Q' is for wall exclusive of veneer. Wall E<sub>7</sub>. Same as E<sub>9</sub> but Q' is for veneer only.



The simplest type of *alignment* chart consists of three parallel scales, so graduated that a straight line joining points on two of the scales will cut the third scale at a point that satisfies the relation between the variables. For example, the expression  $x + y = w$  can be represented

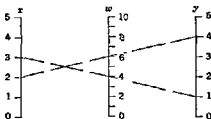


FIG. 17. Alignment Chart for the Equation,  $x + y = w$ .

by the alignment chart shown in Figure 17. The line joining  $x = 3$  with  $y = 1$  cuts the  $w$  scale at point 4, which satisfies the equation  $x + y = w$ . Similarly, the line joining  $x = 2$  with  $y = 4$  yields the value  $w = 6$ .

The simplicity in using alignment charts has won much favor in technology. They, also, are especially advantageous to nontechnical personnel, who can employ them with confidence. Alignment charts are used to show the relationship between three, four, five, or more variables. Some of the charts contain diagonal and curved scales in addition to horizontal and vertical scales. In some cases, it has been found desirable to use a combination of the Cartesian co-ordinate chart and the alignment chart. An example of such a combination is shown in Figure 18—"Nomogram for Determining the Number of Seconds of Green Light for Traffic Signals." Various combinations of alignment charts can be developed. The only limitation is the ingenuity of the designer.

The introductory material that has been presented discloses the fact that nomograms may be of the concurrency (Cartesian co-ordinate) type, or of the alignment form, or of a combination of both. The major portion of the material covered in this book deals with the theory and construction of alignment charts involving straight line scales, curved scales, and combinations of straight line and curved scales. The use of determinants will be demonstrated. However, thorough treatment of the determinant method is not intended since other works which are confined to the "determinants" method only, are available.

Examples of alignment charts which may prove useful in the fields of engineering, production, business, and statistics are included in the Appendix.

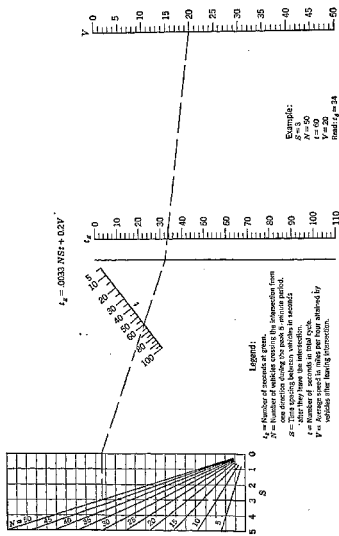


Fig. 18. Combination of Cartesian Co-ordinate Chart and Alignment Chart.

## Chapter Two

### FUNCTIONAL SCALES

The first step necessary in the design of alignment charts is a thorough understanding of the use of functional scales.

A *graphical scale* is a curved or straight line carrying graduations which correspond to a set of numbers arranged in order of magnitude. If the distances between successive points on the scale are equal for equal increments of the variable, the scale is said to be uniform; if not, the scale is *non-uniform*.

A *functional scale* is one on which the graduations are marked with the "values of the variable" and on which the distances to the graduations are laid off in proportion to the corresponding values of the "function of the variable". The distances are laid off from an initial point of the scale, not necessarily the zero point.

#### EXAMPLE

Suppose the function of  $u$ ,  $f(u)$ , is  $u^3$  (Figure 19). Let  $u$  vary from 0 to 5. Form the following table:

$u$	0	1	2	3	4	5
$f(u) = u^3$	0	1	8	27	64	125
$x = u^3$	0	1	8	27	64	125

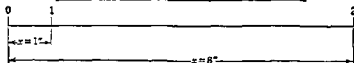


FIG. 19.

We can readily understand that the above scale would be 125 in long if the inch is used as the unit of measure. Obviously this is not a

convenient length. In order to have a scale of a more practical length, a *scale modulus* or scale multiplier is introduced, that is,  $x = mu^3$  or  $m = x/u^3$ , where  $m$  is the scale modulus. The expression  $x = mf(u)$  is called the *scale equation*.

Now suppose that the scale is to be approximately 6 in. long (Figure 20). Then  $m = x/u^3 = 6/125 = 0.048$ . To simplify the computational work,  $m = 0.05$  will be used. This means that the scale length will be 6.25 in. instead of 6 in. The new table then is:

$u$	0	1	2	3	4	5
$f(u) = u^3$	0	1	8	27	64	125
$x = 0.05u^3$	0	0.05	0.40	1.35	3.20	6.25

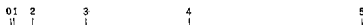


FIG. 20. Functional Scale for  $f(u) = u^3$ .

In the practical use of functional scales further subdivision of the scale into fifths or tenths may be advisable. Suppose that the range of the variable is from 2 to 4 and that the scale length is approximately 6 in. (Figure 21). In this case  $x = m[f(u_2) - f(u_1)] = m(4^3 - 2^3) = 56m$  or  $m = 6/56 = 0.107$ . For convenience we shall use 0.1. Then we have the table:

$u$	2	3	4
$f(u) = u^3$	8	27	64
$x = 0.1(u^3 - 2^3)$	0	1.9	5.6



FIG. 21. Functional Scale for  $f(u) = u^3$ .

It should be noted that in this case the initial point of the scale is 2, not 0.

The distance between any two graduations,  $u_1$  and  $u_2$ , is equal to  $x = m[f(u_2) - f(u_1)]$

Any unit of length other than inches could be adopted as the unit of measure. It is most important to observe (1) that the distance between any two points on the scale is equal to the product of the modulus and the difference in the *values of the function* for the two points, *not the values of the variables*; and (2) that the points are marked with the value of the variable.

Suppose that the  $f(u)$  is  $2u^2$ . The scale equation is  $x = m(2u^2)$ . If  $u$  varies from 0 to 4, and the scale length is to be approximately 6 in., then

$$m = \frac{6}{2(4^2)} = \frac{6}{128} = 0.047$$

For convenience,  $m = 0.05$  will be used. This lengthens the scale to 6.4 in.

Hence,  $x = 0.05(2u^2)$  or  $x = 0.1u^2$ . It is important to note that 0.1 is the *effective modulus*, whereas 0.05 is the actual modulus of the scale. The effective modulus is used in graduating the scale. The "actual modulus" is necessary in the location of scales that occur in alignment chart design. This distinction will be evident later when the design and construction of alignment charts are considered.

If the function of  $u$  is  $(u + 2)$  (Figure 22), then the scale equation is  $x = m(u + 2)$ . If  $u$  varies from 0 to 12, and the scale is 6 in. long, then

$$m = \frac{6 \text{ in.}}{[(12 + 2) - (0 + 2)]} = \frac{6}{12} = \frac{1}{2} \text{ in.}$$

or

$$x = \frac{1}{2}(u + 2)$$

$u$	0	1	2	.	12
$f(u) = (u + 2)$	2	3	4		14
$x = \frac{1}{2}(u + 2)$	1	1.5	2		7

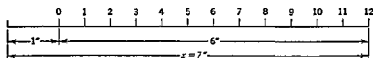


FIG. 22. Scale for the Function,  $(u + 2)$ .

In this case, the constant 2 merely shifts the zero point of the scale a distance from the reference point of the scale equal to  $\frac{1}{2}(0 + 2) = 1$  in. Except for the shift of the zero point, the scale is the same as though the  $f(u)$  were  $u$ , because the total length of the scale is  $x = \frac{1}{2} \times 12 = 6$  in., which is the same as  $x = \frac{1}{2}[(12 + 2) - (0 + 2)] = 6$  in.

## EXAMPLE 1

$$f(u) = \sqrt{u}$$

where  $u$  varies from 0 to 100, scale length approximately 6 in. (Figure 23).

$$m_u = \frac{6}{\sqrt{100} - \sqrt{0}} = \frac{6}{10}$$

$$x_u = 0.6\sqrt{u}$$

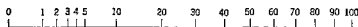


FIG. 23. Functional Scale for  $f(u) = \sqrt{u}$ .

## EXAMPLE 2

$$f(u) = \frac{1}{u^2}$$

where  $u$  varies from 1 to 3, scale length 6 in. (Figure 24).

$$m_u = \frac{6}{\left[\frac{1}{1^2} - \frac{1}{3^2}\right]} = 6.75; \quad x_u = 6.75 \left[\frac{1}{u^2} - \frac{1}{1^2}\right]$$

$u$	1	2	3
$f(u) = \frac{1}{u^2}$	1	$\frac{1}{4}$	$\frac{1}{9}$
$x_u = 6.75 \left[\frac{1}{u^2} - \frac{1}{1^2}\right]$	0	$-\frac{51}{16}$	-6

Note that the negative distances,  $x_u$ , are laid off to the left of point 1, since distances laid off to the right have been regarded as positive.

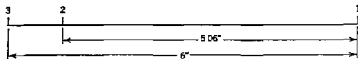


FIG. 24 Functional Scale for  $f(u) = 1/u^2$ .

### EXAMPLE 3

$$f(u) = \log u$$

where  $u$  varies from 2 to 10, scale length, 7 in. (Figure 25).

$$m_u = \frac{7}{\log 10 - \log 2} = \frac{7}{\log 5} = 10$$

$$x_u = 10(\log u - \log 2)$$

A few points are tabulated below:

$u$	2	4	8	10
$\log u$	0.301	0.602	0.903	1.000
$x_u = 10(\log u - \log 2)$	0	3.01	6.03	6.99

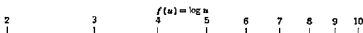


FIG. 25 Functional Scale for  $f(u) = \log u$

#### *Suggestions for Drawing Scales:*

- (1) Make the shortest strokes  $\frac{1}{10}$  in.
- (2) Make the intermediate strokes  $\frac{3}{10}$  in.
- (3) Make the longest strokes  $\frac{3}{16}$  in.
- (4) Adjacent strokes should be not less than  $\frac{1}{20}$  in. apart, or more than  $\frac{1}{2}$  in.

- (5) The distance between labeled strokes should be not less than  $\frac{1}{4}$  in.
- (6) The interval between units should be divided into fifths or tenths, if necessary.

*Suggestions for Graphic Precision:*

- (1) Layout lines should be sharp and light.
- (2) Finish lines should be sharp and dark.
- (3) Use as large a scale as possible.
- (4) If the chart is to be reduced, use a reducing glass to determine the relative weights of lines required in the original drawing.

## EXERCISES

### Functional Scales

- Construct a scale for the function  $f(u) = \sqrt[3]{u}$ .  $u$  varies from 0 to 8. Scale length about 6 in.
- Construct a scale for the function  $f(u) = 2 \log u$ .  $u$  varies from 10 to 300. Scale length about 6 in.
- Construct a scale for the function  $f(u) = \cos u$ .  $u$  varies from  $0^\circ$  to  $90^\circ$ . Scale length 6 in.
- Change the range of the variable of problem 3 to  $0^\circ$  to  $180^\circ$  and construct a scale of the same modulus.
- Construct scales for the functions  $f(u) = \log 1/u$ ,  $\log u$ ,  $\log u^2$ ,  $\log u^3$ .  $u$  varies from 1 to 10. Scale length about 6 in. Write the scale equations for each case.
- Construct a scale for the function  $f(u) = 1/u$ .  $u$  varies from  $\frac{1}{2}$  to  $\infty$ . Scale length about 7 in. Where would be the point,  $u = 0$ ?
- Construct a scale for the function  $f(u) = u^{3/2}$ . Range of the variable 0 to 10. Scale length about 6 in.
- On one side of a line construct a scale for  $w^2$ , on the other side  $u^2 + 3$ . Use the same reference point and same modulus. Range 1 to 5. Do the same for  $\log u$  and  $\log (u + 2)$ . Scale lengths about 6 in.
- Explain the effect of the constant  $K$  on the scales in the following scale equations:

$$x_u = mKu \quad (1)$$

$$x_u = m(u + K) \quad (2)$$

$$x_u = m \log u^K \quad (3)$$

$$x_u = m \log (u + K) \quad (4)$$



10. Construct a scale for the function  $f(t)$  given by the following table, where  $f(t)$  is the vapor pressure in air corresponding to  $t$ , the air temperature, in degrees Fahrenheit:

$t$	$f(t)$	$t$	$f(t)$
0	0.0383	55	0.432
5	0.0491	60	0.517
10	0.0631	65	0.616
15	0.0810	70	0.732
20	0.1026	75	0.866
25	0.130	80	1.022
30	0.164	85	1.201
35	0.203	90	1.408
40	0.247	95	1.645
45	0.298	100	1.916
50	0.360		

11. Given the following experimental data:

$u$	$f(u)$	$u$	$f(u)$
0	0.05	26	0.12
11	0.065	32	0.17
18	0.085	40	0.33

Draw a smooth curve through the points and construct a functional scale for  $f(u)$  from the curve.

12. Construct a scale for the wave length of the various colors of light:

Color	Wave Length
Violet	4000 Angstrom units *
Blue	4500 " "
Green	5200 " "
Yellow	5700 " "
Red	6300 " "

---

\* The Angstrom unit = one ten-millionth of a millimeter, i.e., 0.0000001 mm.

# ADJACENT SCALES FOR THE SOLUTION OF EQUATIONS OF THE FORM

$$f_1(u) = f_2(v)$$

Equations of the above form may be solved by graduating both sides of one line in such a manner that a point on the scale will give values which satisfy the given equation. The scale equations are:

$$X_u = m_u f_1(u) \quad \text{and} \quad X_v = m_v f_2(v)$$

Since  $f_1(u) = f_2(v)$  and  $X_u = X_v$ , i.e., for any point on the scale; then,  $m_u = m_v$ .

## EXAMPLE 1

Consider the relation,  $2.54I = C$  (Figure 26), where  $I$  represents inches and  $C$  represents centimeters. Let  $I$  vary from 0 to 10; scale length, 6 in. If we rewrite the given equation so that

$$I = \frac{C}{2.54}$$

Then,  $X_I = m_I I$ , or  $6 = m_I 10$

from which  $m_I = 0.6$

Thus, the scale equation is  $X_I = 0.6I$ . The scale equation for  $C$  is  $X_C = 0.6(C/2.54) = 0.236C$ .

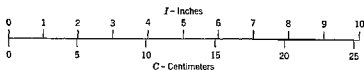


FIG. 26. Adjacent Scales for the Equation,  $2.54I = C$ .

*Note:* Having computed the scale modulus for  $I$ , the same modulus must be used for  $C$ . If the scale modulus for  $C$  had been computed first, then this modulus would apply to  $I$ .

## EXAMPLE 2

$C = \pi D$  (Figure 27).  $C$  = circumference of a circle, with diameter  $D$ . Let  $D$  vary from 2 to 10 in. Scale length, 6 in.

The scale equations are:

$$X_c = m_c \left[ \frac{C}{\pi} \right]$$

and  $X_D = m_D D$ ; or  $6 = m_D[10 - 2]$ ;  $m_D = \frac{3}{4}$

hence  $X_D = \frac{3}{4}[D - 2]$

and  $X_c = \frac{3}{4} \frac{C}{\pi} = \frac{3C}{4\pi} = 0.239C$

To locate a point on the scale  $C$ . For example,  $C = 10$ .

Point 10 on the  $C$  scale is 2.39 in. from the zero point of that scale ( $X_c = 0.239[10]$ ). Since point 2 of the  $D$  scale is  $1\frac{1}{2}$  in., ( $X = \frac{3}{4} \times 2$ ) from the zero point of the  $D$  scale, point 10 is  $(2.39 - 1.50) = 0.89$  in.

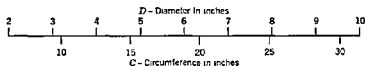


FIG. 27. Adjacent Scales for the Equation,  $C = \pi D$ .

to right of point 2. (It should be observed that  $C = 0$  when  $D = 0$ .) Point 20 of the  $C$  scale is 2.39 in. to the right of point 10 ( $X_c = 0.239 \times [20 - 10] = 2.39$  in.). Points between 10 and 20 can easily be located by subdivision. Points beyond 20 can be located in a similar manner. Of course, a point of the  $C$  scale could be located by solving  $C$  from a specific value of  $D$ . The scale could then be graduated from the scale equation,

$$X_c = 0.239[C_2 - C_1]$$

## EXAMPLE 3

$U = \sin V$  (Figure 28). Let  $V$  vary from  $0^\circ$  to  $90^\circ$ ; scale length, 6 in.

The scale equations are:

$$X_u = m_u U \quad (1)$$

and

$$X_v = m_v \sin V \quad (2)$$

From equation (2)  $6 = m_v(\sin 90^\circ - \sin 0^\circ)$ ;  $m_v = 6$ . Hence the scale equations are:

$$X_u = 6U$$

and

$$X_v = 6 \sin V$$

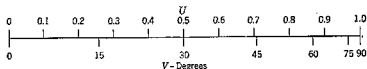


FIG. 28. Adjacent Scales for the Equation,  $U = \sin V$ .

## EXAMPLE 4

$V = \frac{4}{3}\pi r^3$  (Figure 29).  $V$  = the volume of a sphere of radius  $r$ . Let  $r$  vary from 0 to 5 in.; length of scale to be approximately 6 in. Then

$$X_r = m_r r^3; \quad \text{and} \quad X_v = m_v \frac{3V}{4\pi}$$

$$m_r = \frac{6}{125} = 0.048.$$

For convenience, use  $m_r = 0.05$ . Then

$$X_r = 0.05r^3 \quad \text{and} \quad X_v = 0.05 \times \frac{3V}{4\pi} = 0.0119V$$

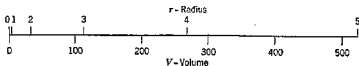


FIG. 29. Adjacent Scales for the Equation,  $V = \frac{4}{3}\pi r^3$ .

# NON-ADJACENT SCALES FOR THE SOLUTION OF EQUATIONS OF THE FORM

$$f_1(u) = f_2(v)$$

In the foregoing material it was pointed out that the same modulus was used in each scale equation. It may be desirable to use two dif-

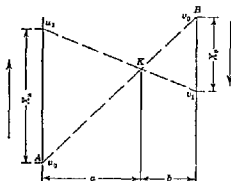


FIG. 30.

ferent moduli. This can be done by separating the scales in the following manner. (See Figure 30.)

$$\text{Let} \quad X_u = m_u f_1(u) \quad (1)$$

$$\text{and} \quad X_v = m_v f_2(v) \quad (2)$$

$u_0$  and  $v_0$  are zero values of the functions of  $u$  and  $v$ ; and  $K$  is a point located on line  $AB$  so that any line passing through  $K$  and a selected point on the  $u$  or  $v$  scale will cut the other scale in a value which satisfies the equation.

From the similar triangles  $Au_1K$  and  $Bv_1K$ ,

$$\frac{X_u}{X_v} = \frac{AK}{KB} = \frac{a}{b}$$

$$\text{Therefore} \quad \frac{m_u f_1(u)}{m_v f_2(v)} = \frac{a}{b} \quad (\text{from equations 1 and 2 above})$$

Since  $f_1(u) = f_2(v)$ ,

$$\frac{a}{b} = \frac{m_u}{m_v}$$

Hence point  $K$  can be located on the diagonal  $AB$  by dividing it into the ratio

$$\frac{AK}{KB} = \frac{a}{b} = \frac{m_u}{m_v}$$

### EXAMPLE

Again consider the equation,  $2.54I = C$  (Figure 31).

Let  $X_I = \frac{1}{2}I$ ;  $X_C = \left(\frac{2.54}{4}\right) \left[\frac{C}{2.54}\right] = \frac{C}{4}$

$$\frac{m_I}{m_C} = \frac{\frac{1}{2}}{\frac{2.54}{4}} = \frac{1}{1.27}$$

### EXERCISES

#### Adjacent Scales or Non-Adjacent Scales

13. Construct (a) adjacent and (b) non-adjacent scales for converting Fahrenheit readings to centigrade.

$$C = \frac{5}{9}(F - 32) \quad C(-40^{\circ} \text{ to } 100^{\circ})$$

14. Construct adjacent scales for the equation area of circle,  $A = \pi d^2/4$ ,  $d(0 \text{ to } 20 \text{ in.})$ . (Do not use log scales.)

15. Use logarithmic scales in problem 14. What are the advantages of each method? Disadvantages?

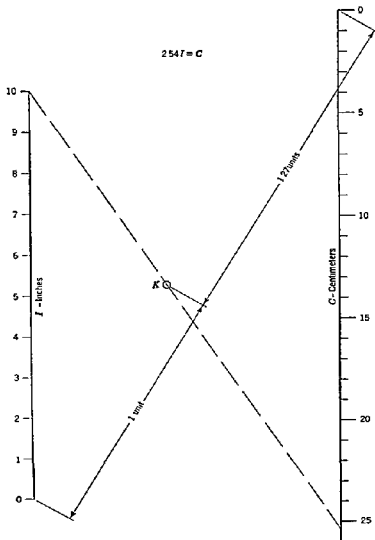
16. Construct adjacent scales for the equation  $t = 2\pi\sqrt{L/g}$ , the period of a simple pendulum of length,  $L$ , and with  $g = 32.2$ .  $L(1 \text{ to } 5 \text{ ft})$ .

17. Construct adjacent scales for the equation,  $X' = \log_e y$ .  $y(1 \text{ to } 10)$ .

18. On a log scale using the base 10, the distance between 1 and 2, 2 and 4, 4 and 8 are equal. Is this true when "e" is used as the base?

19. Construct adjacent scales for the vapor pressure in air as a function of temperature as given in problem 10.

20. Construct non-adjacent scales for the compound interest law,  $P = A(1 + R)^n$ , where  $P$  is the principal,  $R$  the rate of interest,  $A$  the amount, and  $n$  the number of times compounded. Let  $A = \$1.00$ ,  $R = 5\%$ . Then  $P = (1.05)^n$ .  $n$  varies from (0 to 20). How can this chart be used if  $n = 30$ ?

FIG. 31. Separated Scales for the Relation,  $254I = C$ .

## Chapter Three

### ALIGNMENT CHARTS

An alignment chart in its simplest form consists of three parallel scales so graduated that a straight line cutting the scales will determine three points whose values satisfy the given equation.

In general, alignment charts may consist of three or more straight-line scales, of curved scales, or of combinations of both.

#### ALIGNMENT CHARTS FOR EQUATIONS OF THE FORM

$$f_1(u) + f_2(v) = f_3(w)$$

Suppose we have three parallel scales (Figure 32),  $A$ ,  $B$ , and  $C$ , so graduated that lines (isopleths) 1 and 2 cut the scales in values which satisfy the equation  $f_1(u) + f_2(v) = f_3(w)$ . Now,

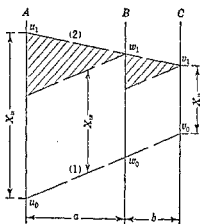


FIG. 32.

$$X_u = m_u[f_1(u_1) - f_1(u_0)]$$

$$X_v = m_v[f_2(v_1) - f_2(v_0)]$$

$$X_w = m_w[f_3(w_1) - f_3(w_0)]$$



If  $u_0, v_0, w_0$ , represent zero values of the functions, and if line 2 is any line, we may drop the subscripts and write simply

$$X_u = m_u f_1(u) \quad (1)$$

$$X_v = m_v f_2(v) \quad (2)$$

$$X_w = m_w f_3(w) \quad (3)$$

Let us agree further that the spacing of the scales is in the ratio  $a/b$ . If we graduate the scales for  $f_1(u)$  and  $f_2(v)$  in accordance with their scale equations (1) and (2), respectively, what will be the modulus for the scale equation of  $f_3(w)$  and what will the ratio  $a/b$  equal if the chart satisfies this relation  $f_1(u) + f_2(v) = f_3(w)$ ?

In Figure 32 draw lines through points  $w_1$  and  $v_1$  parallel to line  $u_0v_0$ . The shaded triangles are similar by construction, hence,

$$\frac{X_u - X_w}{X_w - X_v} = \frac{a}{b}$$

$$X_u b + X_w a = X_w (a + b)$$

$$\frac{X_u b}{ab} + \frac{X_w a}{ab} = X_w \left( \frac{a + b}{ab} \right)$$

or

$$\frac{X_u}{a} + \frac{X_v}{b} = \frac{X_w}{\frac{ab}{a + b}}$$

Since

$$X_u = m_u f_1(u)$$

$$X_v = m_v f_2(v)$$

$$X_w = m_w f_3(w)$$

$$\frac{m_u f_1(u)}{a} + \frac{m_v f_2(v)}{b} = \frac{m_w f_3(w)}{\frac{ab}{a + b}}$$

If  $f_1(u) + f_2(v) = f_3(w)$ , then

$$m_u = a; \quad m_v = b$$

Therefore

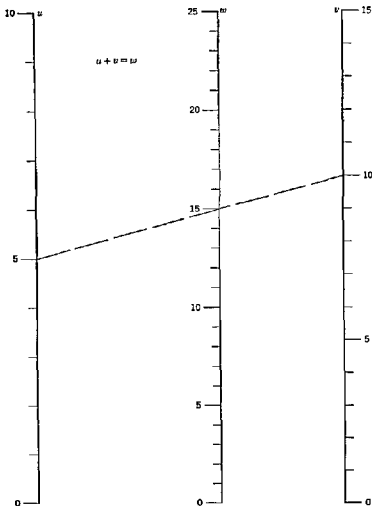
$$\frac{a}{b} = \frac{m_u}{m_v}$$

and

$$m_w = \frac{ab}{a + b} = \frac{m_u m_v}{m_u + m_v}$$

Thus, to construct an alignment chart for an equation of the form,  
 $f_1(u) + f_2(v) = f_3(w)$ ,

- (a) Place the parallel scales for  $u$  and  $v$  a convenient distance apart.
- (b) Graduate them in accordance with their scale equations,  
 $X_u = m_u f_1(u)$  and  $X_v = m_v f_2(v)$ .
- (c) Locate the scale for  $w$  so that its distance from the  $u$  scale is to its distance from the  $v$  scale as  $m_u/m_v = a/b$ .
- (d) Graduate the  $w$  scale from its scale equation,  $X_w = \frac{m_u m_v}{m_u + m_v} f_3(w)$ .

FIG. 33. Alignment Chart for the Equation,  $u + v = w$ .

## EXAMPLE

$u + v = w$  (Figure 33). Let  $u$  vary from 0 to 10; and  $v$  vary from 0 to 15. Suppose that the scale lengths are to be 6 in.

Now  $m_u = \frac{6}{10} = 0.6$ ;  $X_u = 0.6u$

and  $m_v = \frac{6}{15} = 0.4$ ;  $X_v = 0.4v$

and  $m_w = \frac{0.6 \times 0.4}{0.6 + 0.4} = 0.24$ ;  $X_w = 0.24w$

$$\frac{m_u}{m_v} = \frac{0.6}{0.4} = \frac{3}{2} = \frac{a}{b}$$

Now suppose that it is desirable to cut off the chart at the line 1

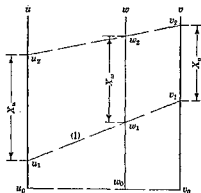


FIG. 34.

(Figure 34), eliminating the portion below line 1. The scale equations will then be (using line 1 as the base line):

$$X_u = m_u[f_1(u_2) - f_1(u_1)]$$

$$X_v = m_v[f_2(v_2) - f_2(v_1)]$$

$$X_w = m_w[f_3(w_2) - f_3(w_1)]$$

where  $u_1$ ,  $v_1$ , and  $w_1$  satisfy the equation  $f_1(u) + f_2(v) = f_3(w)$ .

## EXAMPLE 1

$u + v = w$  (Figure 35). Let  $u$  vary from 2 to 6; and  $v$  vary from 3 to 8. Length of scales, 6 in.

$$\text{Now} \quad m_u = \frac{6}{6-2} = \frac{3}{2}; \quad X_u = \frac{3}{2}(u-2)$$

$$m_v = \frac{6}{8-3} = \frac{6}{5}; \quad X_v = \frac{6}{5}(v-3)$$

$$\text{Since} \quad m_w = \frac{m_u m_v}{m_u + m_v}$$

$$m_w = \frac{\frac{3}{2} \times \frac{6}{5}}{\frac{3}{2} + \frac{6}{5}} = \frac{\frac{18}{10}}{\frac{21}{10}} = \frac{2}{3}$$

Since  $u_1 = 2$  and  $v_1 = 3$ , therefore  $w_1 = 5$ . (From the original equation  $u + v = w$ .) Hence,

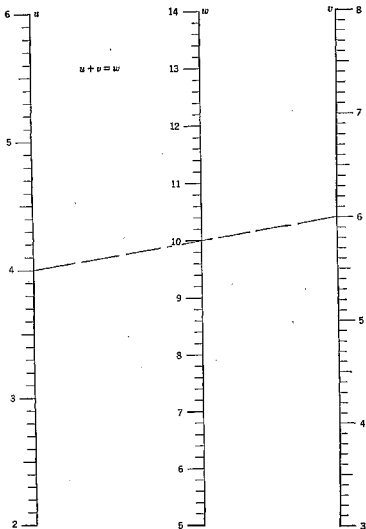
$$X_w = \frac{2}{3}(w-5)$$

$$\frac{a}{b} = \frac{m_u}{m_v} = \frac{\frac{3}{2}}{\frac{6}{5}} = \frac{5}{4}$$

Form the following tables:

$u$	2	3	4	5	6
$f_1(u) = u$	2	3	4	5	6
$X_u = \frac{3}{2}(u-2)$	0	1.5 in	3.0 in	4.5 in	6.0 in

\* If we set  $X_u = m_u f(u)$ . Then we,  $m_w = \frac{X_u}{f_1(u_2) - f_1(u_1)}$ . In this case  $X_u = 6$  in. and  $[f_1(u_2) - f_1(u_1)] = (6-2)$ .

FIG. 35. Alignment Chart for the Equation,  $u + v = w$ .

$v$	3	4	5	6	7	8
$f_2(v) = v$	3	4	5	6	7	8
$X_v = \frac{2}{3}[v - 3]$	0	1.2 in.	2.4 in.	3.6 in.	4.8 in.	6.0 in.

$w$	5	6	7		14
$f_3(w) = w$	5	6	7		14
$X_w = \frac{2}{3}[w - 5]$	0	$\frac{2}{3}$ in.	$\frac{4}{3}$ in.		6 in.

Much of the calculations set forth in the above tables can be eliminated if we compute the location of the end points for each scale, and then project the other points geometrically.

### EXAMPLE 2

$I = \frac{1}{12}bd^3$  (Figure 36), where  $I$  is the moment of inertia of a rectangle about its axis parallel to  $b$ , where  $b$  is the width, and  $d$  is the height of the rectangle.

Let  $b$  vary from 1 to 10 in.,  $d$  from 1 to 10 in. Length of scales, 6 in. The equation, which may be written  $bd^3 = 12I$ , is put in the type form by taking logarithms; thus we obtain

$$\log b + 3 \log d = \log I + \log 12$$

Now the moduli  $m_b$  and  $m_d$  are computed as follows:

$$m_b = \frac{6}{\log 10 - \log 1} = 6; \quad X_b = 6 \log b$$

$$m_d = \frac{6}{3 \log 10 - 3 \log 1} = 2; \quad X_d = 2(3 \log d) = 6 \log d$$

It should be pointed out that the function of  $d$  is  $3 \log d$ , the modulus 2 is the "actual modulus" which is used in locating the  $I$  scale, and the coefficient 6 is the *effective modulus* which is used in graduating the  $d$  scale.

$$m_I = \frac{6 \times 2}{6 + 2} = \frac{12}{8} = \frac{3}{2}; \quad X_I = \frac{3}{2}(\log I + \log 12)$$

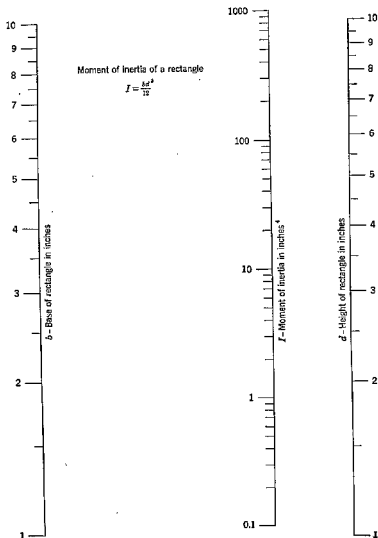


FIG. 30. Moment of Inertia of a Rectangle about Its Axis Parallel to the Base.



Note carefully that the actual moduli of  $b$  and  $d$  are used in computing  $m_I$ . Form the following table:

$b$	1	2	3	...	10
$f(b) = \log b$	0	0.301	0.477		1.000
$X_b = 6 \log b$	0	0.81	2.86		6.00

The table for  $d$  will be the same as the above since  $m_d = 6$  (effective modulus).

Now we can graduate the scales for  $b$  and  $d$  in accordance with the scale equations  $X_b = 6 \log b$  and  $X_d = 2(3 \log d)$ . The scales are placed a convenient distance apart. The position of the  $I$  scale is determined from the ratio  $m_b/m_d = \frac{6}{2} = \frac{3}{1}$ . Our next step is to locate one point on the  $I$  scale, i.e., point 1. Suppose we let  $d = 2$ . Then  $b = 12I/d^3$  or  $b = (12 \times 1)/8 = 1.5$ . The line joining  $b = 1.5$  and  $d = 2$  cuts the  $I$  scale at point 1. Now we can locate other points on the  $I$  scale from the scale equation

$$X_I = \frac{6 \times 2}{6 + 2} [\log I - \log 1]$$

or 
$$X_I = 1.5(\log I)$$

This means that points on the  $I$  scale are laid off from point 1. If the selected point on the  $I$  scale were 10, then graduations would be laid off from this point in accordance with the scale equation:

$$X_I = 1.5[\log I - \log 10]$$

or 
$$X_I = 1.5(\log I - 1)$$

If the equation were  $f_1(u) - f_2(v) = f_3(w)$ , the scale equations would be:

$$X_u = m_u f_1(u) \quad (1)$$

$$X_v = m_v [-f_2(v)] \quad (2)$$

$$X_w = \frac{m_u m_v}{m_u + m_v} f_3(w) \quad (3)$$

The negative sign in equation 2 implies that positive values of  $f_2(v)$  are laid off downwardly if we agree to lay off positive values of  $f_1(u)$  upwardly.

## EXAMPLE

$u - v = w$  (Figure 37). Suppose  $u$  varies from 0 to 5, and  $v$  varies from 2 to 6. Scale lengths, 6 units (inches, centimeters, or any convenient length).

$$m_u = \frac{6}{5}; \quad X_u = \frac{6}{5}u$$

$$m_v = \frac{6}{4} = \frac{3}{2}; \quad X_v = \frac{3}{2}(-v) = -\frac{3}{2}v$$

$$m_w = \frac{\frac{6}{5} \times \frac{3}{2}}{\frac{6}{5} + \frac{3}{2}} = \frac{\frac{9}{5}}{\frac{27}{10}} = \frac{2}{3}; \quad X_w = \frac{2}{3}w$$

$$\frac{m_u}{m_v} = \frac{\frac{6}{5}}{\frac{3}{2}} = \frac{4}{5}$$

Scales  $u$  and  $v$  are placed a convenient distance apart. Scale  $u$  is graduated in accordance with the scale equation  $X_u = \frac{6}{5}u$ . Scale  $v$  is

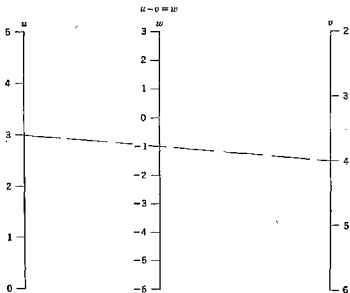


FIG. 37. Alignment Chart for the Equation,  $u - v = w$ .

graduated from the equation  $X_v = \frac{3}{2}v$ . This is done by locating point 2 on the upper end of the  $v$  scale, and laying off distances equal to  $\frac{3}{2}$  units for each point 3, 4, 5, and 6.

NOTE: POSITIVE VALUES OF "V" ARE LAID OFF DOWNWARDLY!

A point on the  $w$  scale can be located by solving the original equation,  $u - v = w$ .

Example: Let  $u = 3$  and  $v = 4$ ; then  $w = -1$  (of course, in this case it might have been simpler to locate the zero point on the  $w$  scale by letting  $u$  and  $v$  equal the same value). Having located one point on  $w$ , other points can be located from the scale equation  $X_w = \frac{2}{3}w$ ; that is, the distance between consecutive points is  $\frac{2}{3}$  units.

## EXERCISES

### Hydraulics

21(a).  $Q = 3.33bh^{3/2}$  (Francis weir formula for a rectangular weir)

where  $Q$  = discharge (cubic feet per second),  $b$  = width of weir (3 to 20 ft),  $H$  = head above crest (0.5 to 1.5 ft).

21(b). Use a double graduation of the  $Q$  scale of part a to indicate gallons per minute.

22.  $V = C\sqrt{2gH}$  (velocity of water, in feet per second, through an orifice due to a head of water,  $H$ )

where  $g = 32.2$ ,  $C$  = a coefficient for the orifice depending on shape, etc. (0.6 to 1), and  $H$  = head of water (1 to 15 ft)

23. The horsepower of a jet of water is given by the equation

$$\text{HP} = \frac{Wv^2}{2g \times 550}$$

where  $W$  = the weight of water per second (1 to 100 lb) and  $v$  = the velocity of water in feet per second (1 to 50).

If desired, since one cubic foot of water is 62.4 lb, the formula may be converted to

$$\text{HP} = \frac{Qv^2 \times 62.4}{2g \times 550}$$

where  $Q$  = the quantity in cubic feet per second and  $g = 32.2$ .

24.  $H = \frac{P}{W}$  (head in feet of a liquid equivalent to the pressure,  $P$ , in pounds per square foot)

where  $W$  is weight per cubic foot of the liquid. If  $P$  is in pounds per square inch,  $H = 144P/W$ .

Let  $P$  vary from 5 to 300 psi. For  $W$  use the common fluids such as water and kerosene.

$$25. \quad H = 0.38 \frac{V^{1.82}}{D^{1.25}} \text{ (friction head for water flowing in 1000 ft of pipe)}$$

where  $V$  = velocity of flow in feet per second (2 to 15) and  $D$  = diameter of pipe in feet (1 to 6).

### Strength of Materials

$$26. \quad R_s = \frac{\pi}{4} d^2 f_s \text{ (allowable strength of a rivet)}$$

where  $d$  = diameter of rivet ( $\frac{1}{8}$  to 1 in.) and  $f_s$  = allowable unit shearing stress of material (3000 to 15,000 psi).

$$27. \quad \rho^2 = \frac{I}{A} \text{ (radius of gyration of a section)}$$

where  $I$  = moment of inertia (1 to 1000 in.<sup>4</sup>) and  $A$  = area of section (1 to 100 sq in.).

$$28. \quad f = \frac{C\pi E}{\left(\frac{L}{P}\right)^2} \text{ (critical stress in a long column)}$$

Let  $C = 1$ , fixity coefficient for pin ended column,  $E$  = modulus of elasticity ( $10 \times 10^6$  to  $30 \times 10^6$  psi), and  $(L/P)$  = slenderness ratio (70 to 200).

$$29. \quad e = \frac{f_x}{E} - \frac{mf_y}{E} \text{ (unit elongation in the } x \text{ direction)}$$

where  $f_x$  = unit stress in  $x$  direction (0 to 50,000 psi),  $f_y$  = unit stress in  $y$  direction (0 to 50,000 psi),  $m = 0.3$ , and  $E = 30 \times 10^6$  psi for structural steel.

### Mechanical

$$30. \quad \text{BHP} = \frac{d^2 n}{2.5} \text{ (Association of Automobile Manufacturers formula)}$$

where  $d$  = diameter of cylinders ( $1\frac{3}{4}$  in. to 5%),  $n$  = number of cylinders (2, 4, 6, 8, 12), and BHP = brake horsepower.

$$31. \quad d = 2.87 \sqrt[3]{\frac{\text{HP}}{\text{RPM}}} \text{ (diameter in inches of a spur gear steel shaft)}$$

where HP = horsepower (100 to 2000) and RPM = revolutions per minute (100 to 1000).

$$32. \quad P_m = \frac{(1 + \log_e R)}{R} P_1 \text{ (absolute mean pressure of expanded steam)}$$

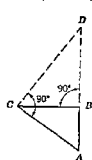
where  $P_1$  = absolute initial pressure (50 to 350 psi) and  $R$  = ratio of expansion =  $V/V_1$  (1 to 10).

$$33. P = CF^{\frac{3}{4}}D^{1\frac{1}{4}} \text{ (pressure in pounds on tool when cutting cast iron)}$$

where  $F$  = feed (0.01 to 0.20 in.),  $D$  = depth of cut ( $\frac{1}{8}$  to 1 in.),  $C$  = 45,000 for soft cast iron, and  $C$  = 69,000 for hard cast iron.

$$34. P = \frac{P_0^{.97} A_0}{60} \text{ (flow of steam through a steam nozzle in pounds per second)}$$

where  $P$  = absolute initial pressure (5 to 300 psi) and  $A_0$  = area at throat (1 to 30 sq in.).



$$35. S = \frac{\pi DN}{12}$$

where  $S$  is the cutting speed in feet per minute in lathe or boring mill,  $D$  = diameter of work (0.25 to 12.5 in.), and  $N$  = (10 to 1000 rpm)

Civil

$$36. C = 0.0000065L(T - T_0) \text{ (change in length of a steel tape due to a difference in temp [} T - T_0 \text{] from the standard temperature, } T_0 \text{)}$$

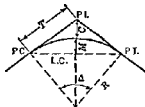
where  $L$  is the measured length (10 to 100 ft), and  $(T - T_0)$  varies from (5° to 75°).

37. In the figure shown it is required to find the inaccessible distance  $BD$ , which is equal to  $BD = BC^2/AB$ .  $BC$  varies (1 to 10) and  $AB$  varies (1 to 10).

38. For the simple curve  $T = R \tan \Delta/2$ , where  $R$  is usually obtained from the degree of curvature and is equal to

$$R = \frac{5729.65}{(\text{degree of curv.})} = \frac{5729.65}{D}$$

where  $D$ , degree of curvature, varies (1° to 30°) and  $\Delta$  varies (10° to 150°).



$$39. L.C. = 2R \sin \frac{\Delta}{2} \text{ (long chord as shown in figure of problem 38)}$$

Use limits of problem 38.

$$40. \quad C = R \operatorname{exsec} \frac{\Delta}{2} \text{ (external distance of a curve)}$$

See figure of problem 38. Use same limits.

$$41. \quad M = R \operatorname{vers} \frac{\Delta}{2} \text{ (midordinate of a curve)}$$

See figure of problem 38. Use same limits.

$$42. \quad L = \frac{\Delta}{D} 100 \text{ (length of the curve)}$$

See figure of problem 38. Use same limits.

$$43. \quad R = \frac{50}{\sin \frac{D}{2}} \text{ (radius of curvature for 100-ft chords)}$$

See figure of problem 38. Use same limits.

$$44. \quad R = \frac{\frac{C}{2}}{\sin \frac{CD}{200}}$$

where it is recommended that

$$C = 100 \text{ ft for } 0^\circ < D < 7^\circ$$

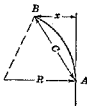
$$C = 50 \text{ ft for } 7^\circ < D < 14^\circ$$

$$C = 25 \text{ ft for } 14^\circ < D < 28^\circ$$

$$C = 10 \text{ ft for } D > 28^\circ$$

45. In a railroad curve as shown the offset  $x$  of  $B$  from the tangent at  $A$  is

$$X = \frac{C^2}{2R}$$



where  $C$  varies (10 to 100 ft) and  $R$  varies (100 to 6000 ft)

## Electrical

$$46. \quad P = \frac{E^2}{R} \quad (\text{power in watts used in passing an electric current through a resistance, } R)$$

where  $E$  = voltage (10 to 220 volts) and  $R$  = resistance (10 to 1000 ohms).

$$47. \quad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad (\text{sum of resistances in parallel})$$

where  $R_1$  (1 to 100 ohms) and  $R_2$  (1 to 100 ohms).

$$48. \quad X = \frac{1,000,000}{2\pi fC} \quad (\text{reactance of a coil})$$

where  $f$  = frequency, cycles per second (30 to 3000) and  $C$  = (1 to 100) capacity of condenser, in microfarads

$$49. \quad E = 0.232 \log \frac{d}{0.78r} \quad (\text{inductive volts per ampere per mile of line with two wires for a 25-cycle current})$$

where  $r$  = the radius of the wire in inches and  $d$  = the spacing in inches (4 to 100). Express  $r$  in  $R$  and  $S$  gage numbers, which vary from (6000 to 10).

50. Double-graduate the  $E$  scale of problem 49 to solve the formula, when using a 60-cycle current:

$$E = (0.232 \times 3.4) \log \frac{d}{0.78r}$$

## Aeronautical

$$51. \quad q = \frac{\rho V^2}{2} \quad (\text{dynamic pressure of air moving at velocity } V)$$

where  $\rho$  = air density slugs per cubic foot (0.0010 to 0.0025) and  $V$  = velocity of air, feet per second (30 to 300).

$$52. \quad R = \frac{S^2}{A} \quad (\text{aspect ratio of wing})$$

where  $S$  = span of airplane wing in feet (20 to 100),  $A$  = area of airplane wing in square feet, and  $R$  = aspect ratio (4 to 7).

53. Since  $V_{\text{mph}} = \frac{22}{7} V_{\text{fps}}$  and 1 slug = 32.2 lb, double-graduate the  $\rho$  and  $V$  scales of problem 51 to increase the usefulness of the chart by eliminating the conversion of units.

$$54. \quad V_s = 20 \left( \frac{S}{C_{L_{\text{max}}}} \right)^{1/2} \quad (\text{stalling speed of an airplane in feet per second})$$

where  $S$  = wing loading, pounds per square foot (5 to 40) and  $C_{L(\max)}$  = maximum lift coefficient (1.1 to 2.5).

$$55. \quad m = m_0 \frac{4}{3 + \frac{6}{R}} \text{ (slope of lift curve for aspect ratio, } R\text{)}$$

where  $m_0$  = slope  $dC_L/d$  at aspect ratio 6, range (4 to 7), and  $R$  = aspect ratio (4 to 7).

56. When the equation of problem 55 is constructed as a  $Z$  type, it is possible by the proper selection of moduli to make all scales uniform. Show how this can be done.

$$57. \quad F_b = \left[ 1.35 - 0.01 \frac{d}{t} \right] \cdot [\text{ult. T.S.}] \text{ (allowable stress in bending in a chrome molybdenum steel tube)}$$

where  $d/t$  = diameter/thickness of wall = thickness ratio (10 to 30) and ult. T.S. = ultimate tensile strength of material (90,000 to 180,000 psi).

### General

$$58. \quad I = \frac{1}{2}Wr^2 \text{ (moment of inertia of a right circular cylinder about its axis)}$$

where  $W$  = total weight in pounds (1 to 25,000) and  $r$  = radius in inches (1 to 25).

$$59. \quad r = \sqrt{x^2 + y^2} \text{ (vector } r \text{ whose co-ordinates are } x \text{ and } y\text{)}$$

where  $x$  varies (0 to 10),  $y$  varies (0 to 10).

$$60. \quad V = 2.467Dd^2 \text{ (volume of a torus)}$$

where  $D$  = larger diameter of torus (1 to 10) and  $d$  = small diameter of torus (1 to 10).

$$61. \quad I = \frac{bd^3}{12} \text{ (moment of inertia of rectangle)}$$

where  $I$  = moment of inertia in inches<sup>4</sup>,  $b$  = width of rectangle (2 to 16 in.), and  $d$  = depth of rectangle (4 to 24 in.).

$$62. \quad P = A(1 + R)^n \text{ (principal } P \text{ after } n \text{ compoundings of the amount } A \text{ at the rate of interest, } R\text{)}$$

Let  $A$  = \$1.00,  $R$  = interest rate in % (1 to 8), and  $n$  = number of times compounded (1 to 20).



## Chemical

63.  $\mu = \mu_1^n$  (specific viscosity referred to water with the same temperature of a salt solution whose normality is  $n$ )

where  $\mu_1$  is the specific viscosity of a normal solution and may be determined from Perry's *Chemical Engineers' Handbook*, p. 679 (1934), for various salt solutions and acids at 25°C,  $n$  varies (0.1 to 1.0), and  $\mu$  varies (0.95 to 1.45).

64.  $V = 174.24\sqrt{t + 459.6}\sqrt{H}$  (velocity of air at or near atmospheric pressure)

where  $V =$  (300 to 15,000 ft per min),  $t =$  temperature in degrees Fahrenheit—range (0° to 1000°), and  $H =$  velocity head in inches (0 to 5).

65.  $V_1 = V\sqrt{\frac{P}{P_1}}$  (corrected velocity of problem 64 for pressures considerably above atmospheric)

where  $V_1 =$  (100 to 10,000 ft per min),  $V =$  as before, problem 64,  $P =$  (14.7 to 100 psi), and  $P_1 = 14.7$  psi.

66.  $V_2 = V_1\sqrt{\frac{1}{S}}$  (correction of  $V_1$  of problem 65 for specific gravity)

where  $V_2 =$  (100 to 20,000 ft per min),  $V_1 =$  (as before), and  $S =$  (0.2 to 1.6), relative to air.

67.  $P = M\left(10.82 - \frac{4425}{T}\right)$  (partial pressure of ammonia in atmosphere over a solution of  $M$  gram moles of ammonia per 1000 grams of water;  $T$  is the temperature in degrees centigrade absolute)

where  $P =$  (0 to 1000 mm),  $M =$  (0 to 100 grams), and  $T =$  (10° to 40°C). Note that the limits of  $P$ ,  $M$ , and  $T$  are not given in the units to be used in the equation. Therefore revise the equation, noting that 1 atmosphere = 760 mm, molecular weight of  $\text{NH}_3 = 17.0$ , and zero degree absolute centigrade = -273°C.

## Statistics

68.  $\sigma_y = \sqrt{\frac{\sum y^2}{N}}$

where  $\sigma_y =$  standard deviation from mean (0.05 to 1000),  $\sum y^2 =$  sum of deviations squared (1 to 10<sup>6</sup>), and  $N =$  number of cases in sample (1 to 1000).

69.  $\sigma_{est} = \sigma_y\sqrt{1 - r_{xy}^2}$  (standard error of estimate in predicting  $y$  scores from  $x$ )

where  $\sigma_y$ , standard deviation of  $y$  scores (1 to 20);  $r_{xy}$ , correlation of  $x$  and  $y$  scores (0.50 to 0.99), and  $\sigma_{est}$ , standard error of estimate (1 to 20).

## Chapter Four

### ALIGNMENT CHARTS (Z CHARTS) FOR THE SOLUTION OF EQUATIONS OF THE FORM

$$f_1(u) = f_2(v) \cdot f_3(w)$$

Suppose that the parallel scales (Figure 38),  $A$  and  $B$ , are graduated in accordance with their scale equations,  $X_u = m_u f_1(u)$  and  $X_v = m_v f_2(v)$ , respectively. The diagonal scale for  $f_3(w)$  joins  $f_1(u_0)$  and  $f_2(v_0)$ , i.e., the zero values of the functions of  $u$  and  $v$ .

Let us suppose further that a straight line joining points  $u_1$  and  $v_1$  cuts

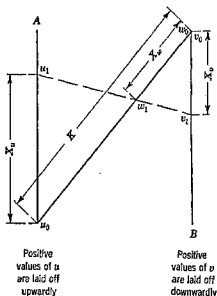


FIG. 38.

the diagonal scale in point  $w_1$  so that the equation  $f_1(u) = f_2(v) \cdot f_3(w)$  is satisfied. What will be the scale equation for  $f_3(w)$ ?

From the similar triangles  $u_0 u_1 w_1$  and  $v_0 v_1 w_1$ ,

$$\frac{X_u}{X_v} = \frac{K - X_w}{X_w}$$

$$\text{or} \quad X_w = X_v \frac{(K - X_w)}{X_w}$$

$$\text{since} \quad X_u = m_u f_1(u)$$

$$\text{and} \quad X_v = m_v f_2(v)$$

$$\text{Then} \quad m_u f_1(u) = m_v f_2(v) \frac{(K - X_w)}{X_w}$$

$$\text{If} \quad f_1(u) = f_2(v) \cdot f_3(w)$$

$$\text{then} \quad \frac{K - X_w}{X_w} = \frac{m_u}{m_v} f_3(w)$$

$$\text{from which} \quad X_w = \frac{K m_v}{m_u f_3(w) + m_v}$$

$$\text{or} \quad X_w = \frac{K}{\frac{m_u}{m_v} f_3(w) + 1} = \frac{K}{K_1 f_3(w) + 1}$$

$$\text{where} \quad K_1 = \frac{m_u}{m_v}$$

If it is desired to graduate the  $w$  scale from  $w_0$  instead of  $v_0$ , it can be shown that the distance from  $w_0$  to  $w_1$  is equal to

$$\frac{K m_u f_3(w)}{m_u f_3(w) + m_v} \quad (\text{This should be verified by the reader.})$$

From the above one can construct this type of chart in the following manner:

- (1) Draw scales for the variables,  $u$  and  $v$ , parallel to each other.
- (2) Graduate the  $u$  scale in accordance with its scale equation,  $X_u = m_u f_1(u)$
- (3) Graduate the  $v$  scale in accordance with its scale equation,  $X_v = m_v f_2(v)$  (plotting positive values of  $v$  downwardly if positive values of  $u$  were plotted upwardly).
- (4) Graduate the  $w$  scale from  $v_0$  in accordance with its scale equation,

$$X_w = \frac{K}{K_1 f_3(w) + 1}, \text{ or from } u_0 \text{ in accordance with the scale equation, } X_w = \frac{K m_u f_3(w)}{m_u f_3(w) + m_v}.$$

## EXAMPLE 1

Consider the equation  $(u + 2) = v^2 w$  (Figure 39). Suppose that  $u$  varies from 0 to 10, and  $v$  from 0 to 5. The scale lengths are to be approximately 6 in.

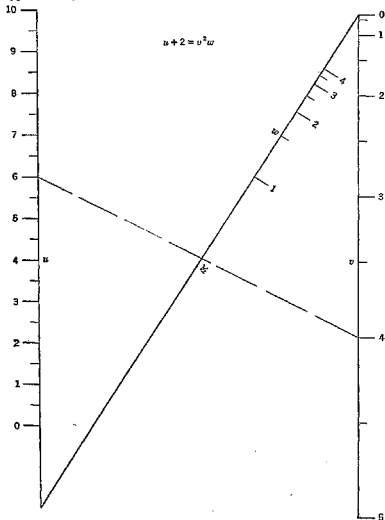


FIG. 39. Z-type Chart for the Equation,  $(u + 2) = v^2 w$ .

$$m_u = \frac{6}{(10+2) - (0+2)} = 0.6$$

$m_v = \frac{6}{25} = 0.24$ . (We shall use 0.25, which merely lengthens the scale from 6 to 6.25 in.)

Hence,  $X_u = 0.6(u+2)$

$$X_v = 0.25v^2$$

and

$$X_w = \frac{10}{\frac{0.6w}{0.25} + 1} = \frac{10}{\frac{12w}{5} + 1}$$

where  $K = 10$  (ten of any convenient unit).

Form the following table:

$w$	0	0.5	1	2	3
$X_w$	10	$\frac{40}{11}$	$\frac{40}{7}$	$\frac{50}{3}$	$\frac{10}{1}$

### EXAMPLE 2

Let us consider the equation for the volume of a right circular cylinder,  $V = \pi r^2 h / 144$ , where  $V$  is the volume in cubic feet,  $r$  is the radius of the base circle, in inches (4 to 12), and  $h$  is the height of the cylinder in feet (4 to 15). We may write the equation,

$$KV = r^2 h, \text{ where } K = \frac{144}{\pi}$$

The range of  $V$  is determined from the ranges of  $r$  and  $h$ . Simple calculations will show that  $V$  varies from 1.40 (or  $4\pi/9$ ) cubic feet to 47.15 (or  $15\pi$ ) cubic feet.

Now

$$m_v = \frac{10 \pm}{\frac{144}{\pi} \left( 15\pi - \frac{4\pi}{9} \right)} = 0.005$$

and

$$X_v = 0.005 \left[ \frac{144}{\pi} \left( v - \frac{4\pi}{9} \right) \right]$$

$$m_r = \frac{10 \pm}{144 - 16} = 0.08$$

$$X_r = 0.08[r^2 - 4^2]$$

From the above scale equations, we can graduate the  $V$  and  $r$  scales. It will be observed that it is necessary only to compute the total length of the  $V$  scale; i.e.,  $X_v = 0.005 \left[ \frac{144}{\pi} \left( 15\pi - \frac{4\pi}{9} \right) \right] = 10.48$ . Then we

know that the lower point of the scale will be marked 1.40 and the upper point will be marked 47.15. Additional graduations can be obtained by proportion. Since the function is linear, the scale is uniform.

In the case of the  $r$  scale, it should be noted that the function is  $r^2$ , and therefore distances between consecutive points are proportional to the square of  $r$ .

The location of the diagonal scale must be determined next. Many students make the typical error of connecting point 4 on the  $r$  scale with point 1.40 on the  $V$  scale. Remember that the diagonal line joins the zero value of the function of  $r$  with the zero value of the function of  $V$ . These points would be zero on the  $r$  scale and zero on the  $V$  scale. In this case, it would be possible to include these points on the respective scales. However, often the zero values of the functions are not accessible within the limits of the drawing. Let us assume this to be the case in our problem.

The position of the  $h$  scale can be established by a very simple method. Let us locate points 6 and 12 on the  $h$  scale. If we let  $r = 10$ , then  $V = 13.1$  when  $h = 6$ . The line joining  $r = 10$  with  $V = 13.1$  contains  $h = 6$ . Again, if we let  $r = 12$ , then  $V = 18.9$  when  $h = 6$ . The line joining  $r = 12$  with  $V = 18.9$  contains  $h = 6$ . Therefore, the intersection of these two lines is  $h = 6$  and, in addition, is a point on the diagonal. This method can be repeated for another point such as  $h = 12$ . Other points on the  $h$  scale can then be located from point 12, by properly using the scale equation,

$$X_h = \frac{K}{\frac{m_v}{m_r} h + 1} = \frac{K}{\frac{0.005}{0.08} h + 1}$$

It is evident that  $K$  must be determined. This can be done, since the distance between points 6 and 12 can be measured. Hence,

$$\frac{K}{\frac{0.005}{0.08} \times 6 + 1} - \frac{K}{\frac{0.005}{0.08} \times 12 + 1} = 2.37$$

from which  $K = 15.23$ .

The distances from  $r = 0$ , along the diagonal, can now be computed from

$$X_h = \frac{15.23}{\frac{0.005}{0.08}h + 1} = \frac{243.68}{h + 16}$$

$h$	4	5	6	7	8	9	10	11	12	13	14	15
$X_h$	12.18	11.59	11.07	10.58	10.13	9.73	9.36	9.01	8.70	8.40	8.12	7.86
$X_h$ from $h = 12$	3.48	2.89	2.37	1.83	1.43	1.03	0.66	0.31	0	0.30	0.58	0.84
← Distances below $h = 12$ →									← Above $h = 12$ →			

The alignment chart is shown in Figure 40.

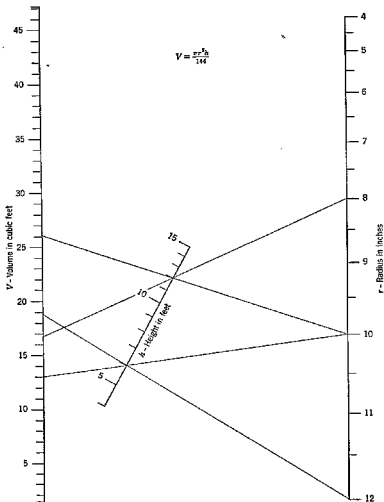


FIG. 40. Z-type Chart for the Relation,  $V = \frac{\pi r^2 h}{144}$ .



## SIMPLIFIED METHOD

A method which may simplify the work of graduating the diagonal scale can be developed in the following manner (Figure 41). Let  $C$  be

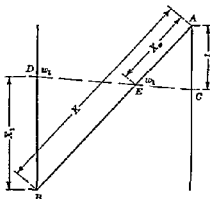


FIG. 41.

a fixed point on the right vertical scale. Let the distance from point  $A$  to the fixed point be  $l$  (inches, centimeters, or any other convenient number of units). Suppose that the *right-hand* side of the left vertical scale carries a temporary  $w$  scale. From the similar triangles,  $BDE$  and  $ACE$ ,

$$\frac{X_1}{l} = \frac{K - X_w}{X_w}$$

$$X_1 = l \left[ \frac{K - X_w}{X_w} \right]$$

Previously it had been shown that

$$\frac{K - X_w}{X_w} = \frac{m_w}{m_v} [f_3(w)]$$

Hence,

$$X_1 = l \frac{m_w}{m_v} [f_3(w)]$$

This equation enables us to graduate the temporary  $w$  scale. Lines joining the fixed point,  $C$ , with the graduations on the temporary scale will intersect the diagonal in points having the same values of  $w$ .

This method has two advantages over the one of locating points on the diagonal from the equation,

$$X_w = \frac{K}{\frac{m_u}{m_v} f_2(w) + 1}$$

First, if the function of  $w$  is linear, a uniform scale can be graduated on the temporary scale; second, the length,  $K$ , of the diagonal scale need not be known.

### EXAMPLE

$$B = \frac{d^2 n}{2.5} \text{ (formula taken from the Association of Automobile Manufacturing)}$$

where  $B$  represents brake horsepower,  $d$  (0 to 5 in.) the diameter of the cylinder in inches, and  $n$  (2, 4, 6, 8, 10, 12) the number of cylinders. The maximum value of  $B = 120$ . Suppose that the lengths of the parallel scales,  $B$  and  $d$ , are 7.5 in. The scale equations will be:

$$X_B = m_B(2.5B); \quad 7.5 = m_B(300); \quad m_B = \frac{7.5}{300} = 0.025$$

$$X_d = m_d(d^2); \quad 7.5 = m_d(25); \quad m_d = \frac{7.5}{25} = 0.3$$

(a) Applying the first method,  $K = 9$  in., we have

$$X_n = \frac{9}{\frac{0.025}{0.3} n + 1} = \frac{9}{\frac{n}{12} + 1} = \frac{108}{n + 12}$$

Form the following table. (Plot  $n$  from these values.)

$n$	2	4	6	8	10	12
$X_n$	7.71	6.76	6.00	5.40	4.91	4.50

(b) Applying the second method, we obtain

$$X_1 = l \frac{m_B}{m_d} n; \quad X_1 = 5 \times \frac{0.025}{0.3} n = \frac{5n}{12}$$

Points on temporary scale  $X_1 = \frac{5}{17}n$  are located on the right-hand side of the  $B$  scale. These points are connected with the fixed point,  $C$ . The intersection of these lines with the diagonal locate the points on the  $n$  scale. In this example,  $l = 5$  in. (See Figure 42 for completed chart.)

### EXERCISES

Since most of the problems of the "three parallel scales" type also fall in the  $Z$  type, construct  $Z$ -type charts for: Problems 22 to 29, inclusive; 31 to 45, inclusive; 47, 50 to 57, inclusive; 60 to 65, inclusive; 68 and 69.

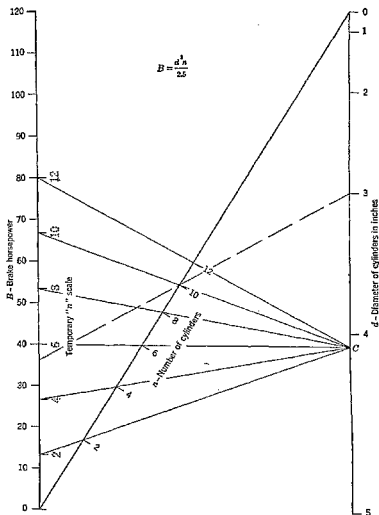


FIG. 42. Z-type Chart, Employing the Simplified Method, for the Expression,

$$B = \frac{d^3 n}{2.5}.$$

Points on temporary scale  $X_1 = \gamma_2^5 n$  are located on the right-hand side of the  $B$  scale. These points are connected with the fixed point,  $C$ . The intersection of these lines with the diagonal locate the points on the  $n$  scale. In this example,  $l = 5$  in. (See Figure 42 for completed chart.)

### EXERCISES

Since most of the problems of the "three parallel scales" type also fall in the  $Z$  type, construct  $Z$ -type charts for. Problems 22 to 29, inclusive; 31 to 45, inclusive; 47; 50 to 57, inclusive; 60 to 65, inclusive; 68 and 69.

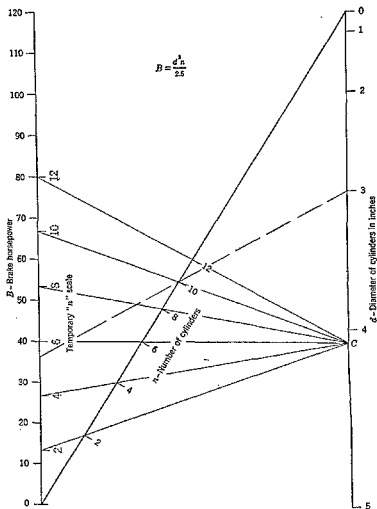


FIG. 42. Z-type Chart, Employing the Simplified Method, for the Expression,

$$B = \frac{d^3 n}{2.5}$$

## Chapter Five

### OTHER FORMS OF EQUATIONS WHICH CAN BE SOLVED BY A Z CHART

$$f_1(u) + f_2(v) = \frac{f_1(u)}{f_3(w)}$$

Let us consider Figure 43, which shows the  $v$  scale on the left, graduated in accordance with the scale equation  $X_v = m_v f_2(v)$  and the  $u$  scale on the right, graduated from the scale equation,  $X_u = m_u f_1(u)$ . Note that positive values of the variable,  $v$ , are plotted upwardly, and that

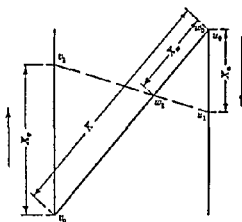


FIG. 43.

positive values of the variable,  $u$ , are plotted downwardly. The diagonal scale which carries the graduations for the function of  $w$  joins the zero values of the functions of  $u$  and  $v$ .

What is the scale equation for the function of  $w$ , in order to have co-linear points on the  $u$ ,  $v$ , and  $w$  scales satisfy the given equation

$$f_1(u) + f_2(v) = \frac{f_1(u)}{f_3(w)}$$

From the similar triangles  $v_0v_1w_1$  and  $u_0u_1w_1$  it follows that

$$\frac{X_v}{X_u} = \frac{K - X_w}{X_w}$$

If  $X_u = m_u f_1(u)$

$$X_v = m_v f_2(v)$$

and  $X_w = m_w f_3(w)$

then  $\frac{m_v f_2(v)}{m_u f_1(u)} = \frac{K - m_w f_3(w)}{m_w f_3(w)}$

or  $\frac{m_u f_1(u) + m_w f_3(w)}{m_u f_1(u)} = \frac{K}{m_w f_3(w)}$

and  $m_u f_1(u) + m_w f_3(w) = \frac{K m_u f_1(u)}{m_w f_3(w)}$

If  $K = m_w$  and  $m_u = m_v$

then  $f_1(u) + f_3(w) = \frac{f_1(u)}{f_3(w)}$

Therefore to construct a chart of this form, graduate the scales in accordance with the scale equations:

$$X_u = m_u f_1(u)$$

$$X_v = m_v f_2(v)$$

$$X_w = K f_3(w)$$

### EXAMPLE

$$f = \frac{20,000}{1 + \frac{144L^2}{9000r^2}} \quad \begin{array}{l} \text{(Gordon Column Formula)} \\ L \text{ (0 to 50 ft)} \\ r \text{ (0 to 10 in.)} \end{array}$$

This equation can be reduced to the form

$$r^2 + 0.016L^2 = \frac{r^2}{\frac{f}{20,000}}$$

The equation is the same form as the type which was developed above.



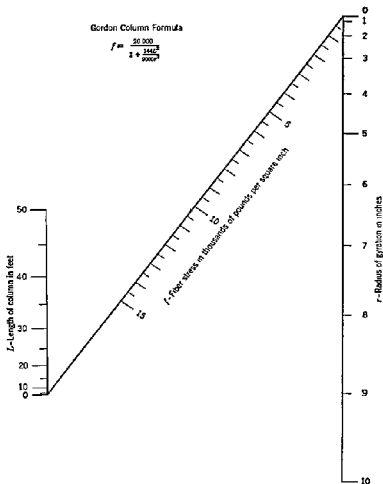


FIG. 44. Gordon Column Formula.

The scale equations are:

$$X_r = m_r r^2$$

$$X_L = m_r (0.016L^2)$$

(Remember that the moduli for the parallel scales are the same.)

$$X_f = K \left( \frac{f}{20,000} \right)$$

If the length of the  $r$  scale is 7.5 in., then

$$m_r = \frac{7.5}{(10)^2} = 0.075$$

or  $X_r = 0.075r^2$

Now  $X_L = 0.075(0.016L^2) = 0.0012L^2$

If we let  $K = 8$

then  $X_f = 8 \left( \frac{f}{20,000} \right) = 0.0004f$

The chart is shown in Figure 44.

If the equation is of the form,

$$f_1(u) - f_2(v) = \frac{f_1(u)}{f_2(w)}$$

the scales for  $u$  and  $v$  will be graduated in the same direction as shown below (Figure 45),

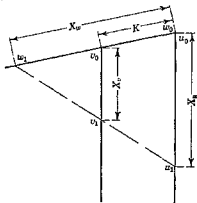


FIG. 45.

## EXERCISES

70. Bazin coefficient for velocity in open channel flow:

$$C = \frac{87}{0.552 + \frac{m}{\sqrt{R}}}$$

where  $m$  = coefficient of roughness (0.06 to 3) and  $R$  = hydraulic radius in feet (0.2 to 25).

71. 
$$\text{S.G.} = \frac{W}{W - W'} \text{ (specific gravity of a body)}$$

where  $W$  = weight in air (0 to 10),  $W'$  = weight in water (0 to 10), and S.G. varies (1 to 15).

72. 
$$\left(\frac{d}{t}\right) = \frac{2f - P}{P}; \left[\left(\frac{d}{t}\right) \text{ required for a thick-walled tube subjected to internal pressure where } \frac{d}{t} \text{ is more than 4}\right]$$

where  $f$  = maximum fiber stress of material (5000 to 60,000 psi),  $P$  = internal pressure (1000 to 10,000 psi), and  $(d/t)$  varies from 4 to 12.

## Chapter Six

### ALIGNMENT CHARTS FOR THE SOLUTION OF EQUATIONS OF THE FORM

$$\frac{1}{f_1(u)} + \frac{1}{f_2(v)} = \frac{1}{f_3(w)}$$

Suppose that the intersecting scales,  $A$  and  $C$ , are graduated in accordance with the scale equations (Figure 46):

$$X_u = m_u f_1(u)$$

$$X_v = m_v f_2(v)$$

If a straight line joining points  $u_1$  and  $v_1$  intersects scale  $B$  in a point

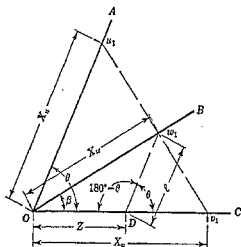


FIG. 46.

$w_1$  so that the given equation is satisfied, how should the  $w$  scale be graduated and how is it located?

Through  $w_1$  let us draw a line parallel to scale  $A$ . From the similar triangles  $Ou_1v_1$  and  $Dw_1v_1$ ,

$$\frac{X_v}{X_u} = \frac{X_v - Z}{\ell}$$

$$X_v \ell = X_u X_v - X_u Z$$

$$\frac{1}{X_u} = \frac{1}{\ell} - \frac{Z}{X_v \ell}$$

$$\frac{1}{m_u f_1(u)} + \frac{Z}{m_v f_2(v) \cdot \ell} = \frac{1}{\ell}$$

$$\frac{1}{f_1(u)} + \frac{Z m_u}{m_v f_2(v) \ell} = \frac{m_u}{\ell}$$

In order that the second term of the left-hand member shall become  $1/f_2(v)$ ,

let 
$$\frac{Z}{\ell} = \frac{m_v}{m_u}$$

from which 
$$\ell = \frac{Z m_u}{m_v}$$

Hence, 
$$\frac{1}{f_1(u)} + \frac{1}{f_2(v)} = \frac{m_v}{Z}$$

If 
$$Z = m_v f_3(w)$$

then 
$$\frac{1}{f_1(u)} + \frac{1}{f_2(v)} = \frac{1}{f_3(w)}$$

Therefore, to construct a chart of the above form:

(a) Graduate the scales for  $f_1(u)$  and  $f_2(v)$  in accordance with their scale equations,  $X_u = m_u f_1(u)$  and  $X_v = m_v f_2(v)$ , respectively.

(b) Locate the scale for  $f_3(w)$  so that

$$\frac{Z}{\ell} = \frac{m_v}{m_u}$$

(c) Graduate the  $C$ -scale (see Figure 46) with a temporary  $w$  scale, using the scale equation,  $Z = m_v f_3(w)$ , and project the points of this scale onto the  $w$  scale by means of parallels to the scale  $A$ .

## ALTERNATE METHOD

A method for graduating the  $w$  scale directly can be developed in the following manner:

By trigonometry,

$$\begin{aligned}
 X_w^2 &= Z^2 + \ell^2 - 2Z\ell \cos (180^\circ - \theta) \\
 &= Z^2 + \left( Z \frac{m_u}{m_v} \right)^2 - 2Z^2 \left( \frac{m_u}{m_v} \right) (-\cos \theta) \\
 &= Z^2 \left[ 1 + \left( \frac{m_u}{m_v} \right)^2 + 2 \left( \frac{m_u}{m_v} \right) \cos \theta \right] \\
 &= \left[ m_v^2 \cdot f_3(w)^2 \right] \left[ 1 + \frac{m_u^2}{m_v^2} + 2 \frac{m_u}{m_v} \cos \theta \right] \\
 &= [m_v^2 + m_u^2 + 2m_um_v \cos \theta] f_3(w)^2 \\
 X_w &= [m_v^2 + m_u^2 + 2m_um_v \cos \theta]^{1/2} f_3(w)
 \end{aligned}$$

Then let  $X_w = m_w f_3(w)$

Therefore  $m_w = [m_v^2 + m_u^2 + 2m_um_v \cos \theta]^{1/2}$

The  $w$  scale can be located by the relation

$$\frac{Z}{\ell} = \frac{m_v}{m_u}$$

## EXAMPLE

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \quad (\text{lens formula}) \quad (\text{Figure 47})$$

where  $u$  = object distance (0 to 100),  $v$  = image distance (0 to 80),  
 $f$  = focal distance (0 to 50).

The scale equations are:

$$X_u = m_u(u); \quad m_u = \frac{6}{100}; \quad X_v = 0.06u$$

where the length of the  $u$  scale is 6 in.

$$X_v = m_v(v); \quad m_v = \frac{4}{80}; \quad X_v = 0.05v$$

where the length of the  $v$  scale is 4 in.

$$\text{Let} \quad \theta = 60^\circ$$

$$\begin{aligned} m_f &= (m_v^2 + m_u^2 + 2m_um_v \cos \theta)^{1/2} \\ &= (0.06^2 + 0.05^2 + 2 \times 0.06 \times 0.05 \times 0.5)^{1/2} \\ &= (0.0036 + 0.0025 + 0.003)^{1/2} \\ &= 0.0955 \end{aligned}$$

$$X_f = 0.0955f$$

$$\frac{Z}{\ell} = \frac{0.05}{0.06} = \frac{5}{6} \quad \left( \text{Recall that } \frac{Z}{\ell} = \frac{m_u}{m_v} \right)$$

Only one point, such as point 50, need be located on the  $f$  scale from the scale equation  $X_f = 0.0955f$ . The other points may be projected geometrically. The same is true of the  $u$  and  $v$  scales.

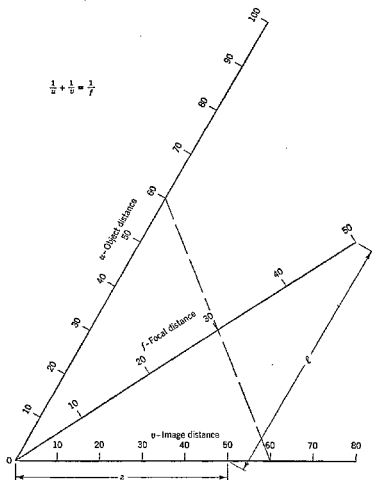


FIG. 47. Alignment Chart for the Relation,  $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ .



## SPECIAL CASE 1

If  $m_v = m_u$  and  $\theta = 120^\circ$ ,

$$X_w = m_w f_3(w) = m_u f_3(w)$$

$$\frac{Z}{\ell} = \frac{m_v}{m_u} = 1$$

It can be shown easily by geometry that whenever

$$\frac{Z}{\ell} = 1$$

the  $w$  scale bisects the angle  $\theta$ ; that is,  $\beta = \theta/2$  (see Figure 48).

$$\begin{aligned} m_w &= [m_u^2 + m_u^2 + 2m_u^2(-\frac{1}{2})]^{1/2} \\ &= m_u \end{aligned}$$

Hence the three scales would have the same modulus.

## EXAMPLE

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{1}{r} \quad (\text{Figure 48})$$

Let  $r_1$  and  $r_2$  vary from 0 to 10 ohms, and  $r$  from 0 to 5 ohms. If the scale lengths for  $r_1$  and  $r_2$  are  $2\frac{1}{2}$  in., then

$$m_{r_1} = \frac{2.5}{10} = 0.25$$

$$m_{r_2} = m_{r_1} = 0.25$$

Therefore, the scale equations are:

$$X_{r_1} = 0.25r_1 \quad \text{and} \quad X_{r_2} = 0.25r_2$$

Since the angle between the scales is  $120^\circ$ ,  $m_r = m_{r_1} = 0.25$ . Thus the scale equation for the  $r$  scale is  $X_r = 0.25r$ . Again, since  $m_{r_2} = m_{r_1}$ , the  $r$  scale bisects the angle between the  $r_1$  and  $r_2$  scales. The completed chart is shown in Figure 48.

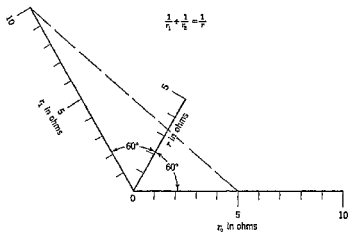


FIG. 48. Alignment Chart for the Equation,  $\frac{1}{r_1} + \frac{1}{r_2} = \frac{1}{r}$ .

### SPECIAL CASE 2

If  $\theta = 90^\circ$ , we have

$$m_w = [m_u^2 + m_v^2 + 2m_um_v \cos 90^\circ]^{1/2}$$

$$m_w = [m_u^2 + m_v^2]^{1/2}$$

$$\frac{Z}{l} = \frac{m_v}{m_u} = \cot \beta$$

$$\beta = \cot^{-1} \left( \frac{m_v}{m_u} \right)$$

## EXAMPLE

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R} \quad (\text{Figure 49})$$

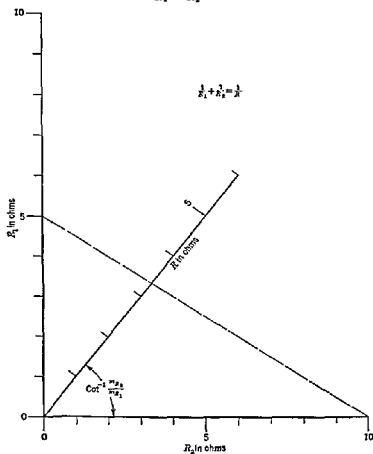


FIG. 49. Alignment Chart for the Equation,  $\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R}$ . Two Scales at Right Angles.

Let  $R_1$  and  $R_2$  vary from 0 to 10 ohms, and  $R$  from 0 to 5 ohms. Suppose that the  $R_1$  scale is 4 in. long; that the  $R_2$  scale is 3 in. long; and that the angle between the scales is  $90^\circ$ . Now

$$m_{R_1} = \frac{4}{10} = 0.4$$

$$X_{R_1} = 0.4R_1$$

Furthermore,

$$m_{R_2} = \frac{3}{10} = 0.3$$

and

$$X_{R_2} = 0.3R_2$$

From the above, it follows that

$$m_R = [0.4^2 + 0.3^2]^{\frac{1}{2}} = 0.5$$

Therefore

$$X_R = 0.5R$$

The  $R$ -scale is located by the ratio  $m_{R_2}/m_{R_1} = 0.3/0.4$ .

### EXERCISES

$$73. \quad \frac{1}{u^2} + \frac{2}{v} = \frac{1}{w^3}$$

where  $u$  varies 0 to 5 and  $v$  varies 0 to 10.

$$74. \quad \frac{1}{u} + \frac{3}{v^2} = \frac{1}{w}$$

where  $u$  varies 0 to 10 and  $v$  varies 0 to 8.

## Chapter Seven

### ALIGNMENT CHARTS FOR EQUATIONS OF FOUR OR MORE VARIABLES OF THE FORM

$$f_1(u) + f_2(v) + f_3(w) \cdots = f_4(q)$$

#### EXAMPLE 1

Let us consider the relation

$$u + 2v + 3w = 4t$$

$$\text{Let} \quad u + 2v = Q \quad (1)$$

$$\text{then} \quad Q + 3w = 4t \quad (2)$$

These two equations are of the form discussed in Chapter Three.

Suppose (equation 1) that  $m_u = 1$ ;  $m_v = \frac{1}{2}$ ; then

$$X_u = u \quad \text{and} \quad X_v = \frac{1}{2}(2v) = v$$

$$\text{Now} \quad \frac{m_u}{m_v} = \frac{1}{\frac{1}{2}} = \frac{2}{1}; \quad m_Q = \frac{1 \times \frac{1}{2}}{1 + \frac{1}{2}} = \frac{1}{3}$$

$$\text{If (equation 2)} \quad m_w = \frac{1}{3}, \quad X_w = \frac{1}{3}(3w) = w$$

$$\frac{m_Q}{m_w} = \frac{\frac{1}{3}}{\frac{1}{3}} = \frac{1}{1}; \quad m_t = \frac{\frac{1}{3} \times \frac{1}{3}}{\frac{1}{3} + \frac{1}{3}} = \frac{\frac{1}{9}}{\frac{2}{3}} = \frac{1}{6}$$

$$\text{Therefore} \quad X_t = \frac{1}{6}(4t) = \frac{2}{3}t$$

From the above calculations, we may now proceed to construct the chart (Figure 50).

Now consider *equation 1*,  $u + 2v = Q$ . Scales  $u$  and  $v$  are placed a convenient distance apart. Scale  $u$  is graduated from its scale equation,  $X_u = u$ ; and scale  $v$  is graduated from its scale equation,  $X_v = \frac{1}{2}(2v) = v$ . The  $Q$  scale is located in accordance with the ratio,  $m_u/m_v = 1/\frac{1}{2} = 2/1$ . This scale is *not* graduated.

Now consider *equation 2*,  $Q + 3w = 4t$ . Scale  $w$  is placed a convenient distance from the  $Q$  scale. Graduations on the  $w$  scale are located in accordance with its scale equation,  $X_w = \frac{1}{3}(3w) = w$ . The  $t$  scale

is located from the ratio,  $m_Q/m_w = \frac{1}{3}/\frac{1}{3} = 1/1$ . The  $t$  scale is then graduated from the scale equation,  $X_t = \frac{1}{4}(4t) = \frac{2}{3}t$ .

It should be carefully noted that in most of the practical applications of this form, it is necessary to locate a point on the fourth scale (by a

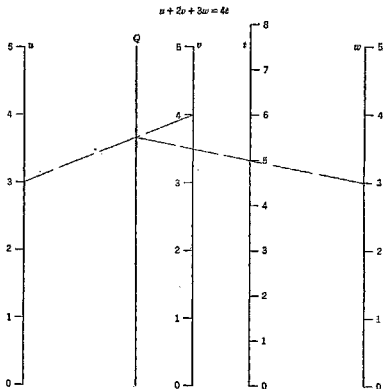


FIG. 50. Alignment Chart for the Equation,  $u + 2v + 3w = 4t$ .

computation from the given equation) before graduating that scale. The chart is shown in Figure 50.

## EXAMPLE 2

$$R = 19.64Cd^2\sqrt{h}$$

where  $R$  = rate of flow through an orifice, in gallons per minute;  $C$  = orifice coefficient (0.6 to 1.6);  $d$  = orifice diameter (0.1 to 1.0 in.), and  $h$  = head (10 to 100 ft).

Our first step is to write the given equation in type form:

Case a:

$$\log R - \log 19.64 = \log C + 2 \log d + \frac{1}{2} \log h$$

$$\text{Let} \quad \log C + 2 \log d = T \quad (1)$$

$$\text{and} \quad \log R - \log 19.64 = T + \frac{1}{2} \log h \quad (2)$$

Equations 1 and 2 are now of the form  $f_1(u) + f_2(v) = f_3(w)$ . The nomogram would look like Figure 51.

With this arrangement of scales, it will be observed that the operation of the chart would require, first, a line (isopleth) joining points on

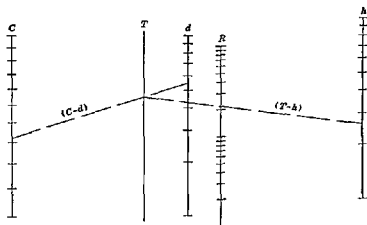


FIG. 51.

the  $C$  and  $d$  scales. The intersection of this line with the  $T$  scale (dummy or turning axis) would then be joined with a point on the  $h$  scale. The intersection of the latter line with the  $R$  scale would give the result.

Now let us try a different analysis of the problem. Suppose we write the equations:

Case b:

$$\log C + 2 \log d = T$$

and  $\log R - \log 19.64 - \frac{1}{2} \log h = T$

Now the arrangement of scales would look like Figure 52. *Note:* Graduations on the  $h$  scale will be directed downwardly, since a minus sign precedes  $\frac{1}{2} \log h$ .

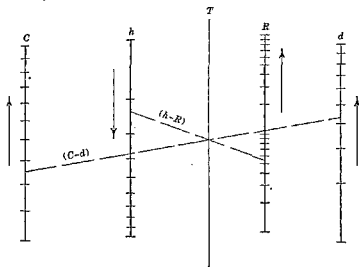


FIG. 52.

Case c:

A third analysis of the given equation shows that we could write:

$$\log R - \log 19.64 = T + \frac{1}{2} \log h$$

and  $T - \log C = 2 \log d$

The arrangement of scales in this case would look like Figure 53. Note that this arrangement is much better than the first two, since there is greater clarity in operation and reading.

Having completed the preliminary studies, let us now make the



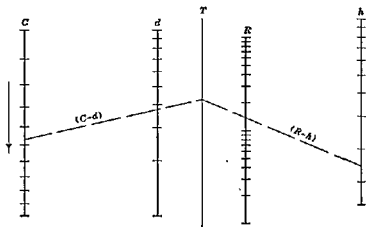


FIG. 53

necessary computations for the final design of the chart which is shown above. Consider equation  $T - \log C = 2 \log d$ .

For scale  $C$

$$m_C = \frac{X_C}{[\log C_n - \log C_0]} = \frac{4 \text{ in. } \pm}{\log 1.6 - \log 0.6} \approx 9.4 \text{ (use 10.0)}$$

or  $X_C = 10[\log C - \log 0.6]$ , scale equation for  $C$

For scale  $d$

$$m_d = \frac{4 \text{ in. } \pm}{2 \log 1.0 - 2 \log 0.1} = 2$$

$$X_d = 2[2 \log d - 2 \log 0.1], \text{ scale equation for } d$$

Now 
$$m_d = \frac{m_C \cdot m_T}{m_C + m_T}$$

or 
$$2 = \frac{10m_T}{10 + m_T}$$

$$m_T = 2.5; \quad \frac{m_C}{m_T} = \frac{10}{2.5} = \frac{4}{1}$$

Sketch layout for  $T - \log C = 2 \log d$  is shown in Figure 54. Now let us consider the equation

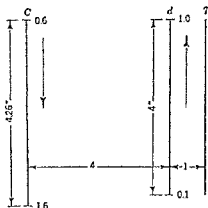


FIG. 54.

$$\log R - \log 19.64 = T + \frac{1}{2} \log h$$

For scale  $h$

$$m_h = \frac{7.5 \text{ in.}}{\frac{1}{2} \log 100 - \frac{1}{2} \log 10} = 15$$

or  $X_h = 15[\frac{1}{2} \log h - \frac{1}{2} \log 10] = 7.5[\log h - \log 10]$

$$\frac{m_T}{m_h} = \frac{2.5}{15} = \frac{1}{6}$$

The modulus for the  $R$  scale is

$$m_R = \frac{m_T \cdot m_h}{m_T + m_h} = \frac{2.5 \times 15}{2.5 + 15} = 2.14$$

and  $X_R = 2.14[\log R - \log R_1]$

where  $R_1$  is a point on the  $R$  scale. This point is computed from the original equation.

The sketch layout for  $\log R - \log 19.64 = T + \frac{1}{2} \log h$  is shown in Figure 55. The completed chart is shown in Figure 56.

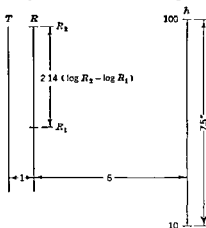
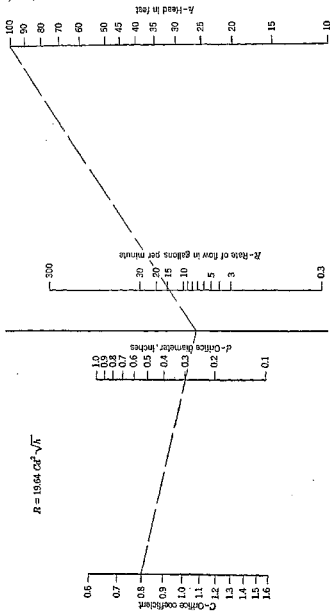


FIG 55.

The designer is cautioned to check the positioning of the  $R$  scale in each case, before the adoption of the final form. In some cases, it will be found that the most desirable form, case  $c$  in the above example, may yield one scale whose graduations are not properly oriented with respect to the other scales, that is, one scale may be practically out of reach in spite of the fact that the "length" of the graduated scale is satisfactory.

FIG. 56. Alignment Chart for the Expression,  $R = 19.64 C d^2 \sqrt{h}$ .

## Chapter Eight

### PROPORTIONAL CHARTS OF THE FORM

$$\frac{f_1(u)}{f_2(v)} = \frac{f_3(w)}{f_4(q)}$$

This equation can be solved in a manner similar to that used in the preceding problem. This simply means transforming the above equation to the form

$$\log f_1(u) - \log f_2(v) = \log f_3(w) - \log f_4(q)$$

In many cases, however, where the functions are linear, the proportional type alignment chart has a distinct advantage in that the scales

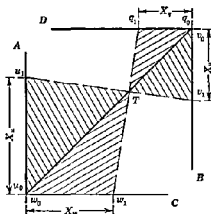


FIG 57.

are uniform, thus permitting more accurate readings and also simplifying the construction of the scales.

Consider the figure shown in Figure 57.

Scales A and B are parallel to each other, and graduated in accordance with the scale equations:

$$X_u = m_u f_1(u) \quad \text{and} \quad X_v = m_v f_2(v), \text{ respectively}$$

In a similar manner, scales  $C$  and  $D$  are parallel to each other, and are graduated in accordance with the scale equations:

$$X_w = m_w f_3(w) \quad \text{and} \quad X_q = m_q f_4(q)$$

The angle between scales  $A$  and  $C$  may be of any convenient magnitude.

Triangles  $u_1 u_0 T$  and  $v_1 v_0 T$  are similar, hence

$$\frac{X_u}{X_v} = \frac{u_0 T}{v_0 T}$$

Likewise, triangles  $w_0 T w_1$  and  $q_0 T q_1$  are similar, hence

$$\frac{X_w}{X_q} = \frac{w_0 T}{q_0 T}$$

But lengths  $u_0 T = w_0 T$ ; and  $v_0 T = q_0 T$ . Therefore

$$\frac{X_u}{X_v} = \frac{X_w}{X_q}$$

or

$$\frac{m_u f_1(u)}{m_v f_2(v)} = \frac{m_w f_3(w)}{m_q f_4(q)}$$

Since

$$\frac{f_1(u)}{f_2(v)} = \frac{f_3(w)}{f_4(q)}$$

it follows that

$$\frac{m_u}{m_v} = \frac{m_w}{m_q}$$

This means that three moduli may be determined from the given data, but the fourth modulus will be dependent upon the first three.

## EXAMPLE 1 (Figure 58)

$t = \frac{Pd}{2f}$  (thickness of a pipe to withstand internal pressures, where  $P$  = pressure (25 to 100 psi)

where  $f$  = allowable stress (3000 to 15,000 psi),  $d$  = diameter of pipe (10 to 60 in.), and  $t$  = thickness of pipe ( $\frac{1}{8}$  to  $\frac{1}{2}$  in.). The given equation may be put in type form by writing  $\frac{t}{d} = \frac{P}{2f}$ .

$$t = \frac{Pd}{2f}$$

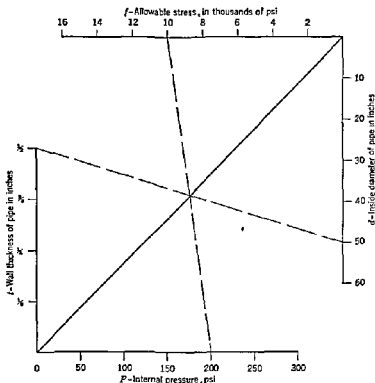


FIG 58 Proportional Type Chart for the Relation,  $t = \frac{Pd}{2f}$ .

Now  $m_t = \frac{5 \text{ in.}}{\frac{1}{2}} = 10 \text{ in.}$

and  $X_t = 10t$  (scale equation for  $t$ )

$$m_d = \frac{5 \text{ in.} \pm}{60} = 0.0833 \text{ (use 0.1)}$$

and  $X_d = 0.1d$  (scale equation for  $d$ )

$$m_f = \frac{5 \text{ in.} \pm}{30,000} = 0.000166 \text{ (use 0.0002)}$$

and  $X_f = 0.0002(2f) = 0.0004f$

The modulus for  $P$  is now computed from  $m_t/m_d = m_p/m_f$ ; or  $10/0.1 = m_p/0.0002$ ;  $m_p = 0.02$  and  $X_p = 0.02P$ . (See Figure 58 for completed chart.)



## EXAMPLE 2

$$E = 15(V - v) \left( 1 + \frac{w}{10} \right) \text{ (Meyer's evaporation formula) (Figure 59)}$$

where  $E$  = the evaporation in inches per month (0 to 10);  $V$  = saturated vapor pressure corresponding to monthly mean temperature,  $t$ , degrees Fahrenheit, which varies from  $30^\circ$  to  $90^\circ$ ;  $v$  = the actual vapor pressure;  $w$  = the monthly mean wind velocity, mph (0 to 30).

Since

$$v = V \times \text{R.H.}$$

where R.H. = the monthly mean relative humidity (30% to 90%) and  $E = 15V(1 - \text{R.H.})(1 + w/10)$ .

$$\text{Or} \quad \frac{E}{10 + w} = \frac{1.5(1 - \text{R.H.})}{\frac{1}{f(t)}}$$

where  $V$  is a function of  $t$ .

$$m_E = \frac{6.25}{10} = \frac{5}{8}; \quad X_E = \frac{5}{8}E$$

Range of  $f(t)$  is 0.164 to 1.408.

$$m_t = \frac{4\frac{1}{2}}{1} = 0.738 \text{ (use 0.75); } X_t = 0.75 \frac{1}{f(t)}$$

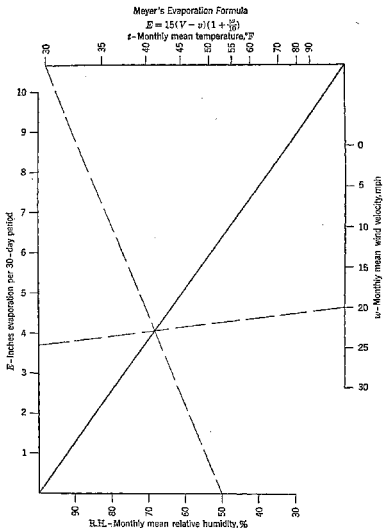


FIG. 59. Meyer's Evaporation Formula,  $E = 15(V - v)\left(1 + \frac{w}{10}\right)$ .

$$m_w = \frac{5}{10 + 30} = \frac{1}{8}; \quad X_w = \frac{1}{8}(10 + w)$$

$$\frac{\frac{5}{8}}{\frac{1}{8}} = \frac{m_{R.H.}}{0.75}$$

$$\begin{aligned} \text{Therefore } m_{R.H.} &= \frac{1.5}{4}; \quad X_{R.H.} = \frac{1.5}{4} \times 1.5(1 - \text{R.H.}) = \frac{9}{16}(1 - \text{R.H.}) \\ &= \frac{1.5}{6}(1 - \text{R.H.}) \end{aligned}$$

The angle between scales *A* and *C* (or *D* and *B*) need not be  $90^\circ$ . In fact, scale *C* could coincide with scale *A*, which means that scale *D*

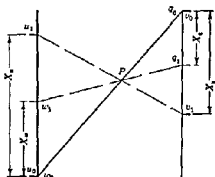


FIG. 60.

would coincide with scale *B*. A study of Figure 60 will reveal that the above statement is true.

$$\frac{X_u}{X_w} = \frac{u_0 P}{P q_0}$$

$$\frac{X_w}{X_u} = \frac{w_0 P}{P q_0}$$

and since  $u_0 = w_0$  and  $r_0 = q_0$ ,

$$\frac{X_u}{X_w} = \frac{X_w}{X_u}$$

Hence, if

$$X_u = m_u f_1(u)$$

$$X_v = m_v f_2(v)$$

$$X_w = m_w f_3(w)$$

$$X_q = m_q f_4(q)$$

and

$$\frac{m_u}{m_v} = \frac{m_w}{m_q}$$

then

$$\frac{f_1(u)}{f_2(v)} = \frac{f_3(w)}{f_4(q)}$$

Variations of the above charts are shown in Figures 61, 62, and 63.

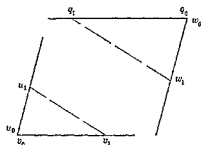


FIG. 61.

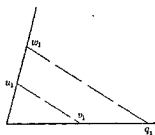


FIG. 62.

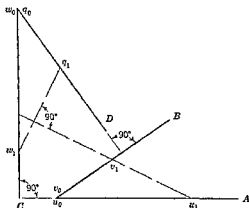


FIG. 63.

## EXERCISES

## Hydraulics

$$75. \quad t = \frac{Pd}{2f} \text{ (required thickness of a pipe to withstand an internal pressure)}$$

where  $t$  = thickness in inches (0 to  $\frac{1}{4}$ );  $P$  = internal pressure, pounds per square inch (0 to 100);  $f$  = allowable stress, pounds per square inch (0 to 15,000);  $d$  = diameter of pipe in inches (0 to 60).

$$76. \quad R = \frac{\rho v l}{\mu} \text{ (Reynolds number as used for fluid motion (0 to 1,000,000))}$$

where  $v$  = velocity, feet per second (0 to 10);  $l$  = characteristic dimension (0 to 2 ft);  $\mu$  = coefficient of viscosity, and for water is a function of temperature given by the following table:

$T$	$\mu$
32°F	$374 \times 10^{-7}$
50	$273 \times 10^{-7}$
68	$211 \times 10^{-7}$
86	$167 \times 10^{-7}$
104	$137 \times 10^{-7}$
122	$115 \times 10^{-7}$
140	$97.8 \times 10^{-7}$
158	$84.6 \times 10^{-7}$
176	$74.4 \times 10^{-7}$
194	$66.1 \times 10^{-7}$
212	$59.2 \times 10^{-7}$

$$\rho = \frac{62.4}{32.2} \text{ (for water)}$$

$$77. \quad Q = C_d a \sqrt{2gh} \text{ (discharge from an orifice or nozzle in cubic feet per second (0 to 20))}$$

where  $C_d$  = coefficient of discharge (0.5 to 1.0);  $a$  = area of the orifice (0 to 1.0 sq ft);  $h$  = head of water on the orifice (0 to 20 ft),  $g = 32.2$

$$78. \quad v = \frac{1.486}{n} R^{2/3} S^{1/2} \text{ (velocity in an open channel (0 to 30 ft per second) [Manning's formula])}$$

where  $n$  = coefficient of roughness (0.002 to 0.035);  $R$  = hydraulic radius (0 to 20 ft);  $S$  = slope of channel (0 to 0.01).

$$79. \quad V = C \sqrt{RS} \text{ (Chezy formula for velocity in an open channel)}$$

Use same limits as in problem 78.

$$80. \quad \frac{N_s}{N} = \sqrt{\frac{HP}{H^3}} \text{ (formula for a reaction turbine)}$$

where  $N_s$  = specific speed (10 to 100);  $N$  = speed in rpm (100 to 2000);  $HP$  = horsepower (to 1000);  $H$  = head of water (10 to 200 ft).

### Strength of Materials

$$81. \quad f = \frac{6M}{bH^2} \text{ (Stress in the outer fiber of a section of a rectangular beam)}$$

where  $M$  = bending moment on the section in inch-pounds (10,000 to 300,000);  $b$  = breadth of section, inches (2 to 16);  $h$  = depth of section, inches (3 to 20);  $f$  = fiber stress, pounds per square inch (750 to 1300).

$$82. \quad f = \frac{Mc}{I} \text{ (fiber stress in a beam of any cross section)}$$

where  $f$  = fiber stress, pounds per square inch (3000 to 15,000);  $M$  = bending moment on section in inch-pounds (25,000 to 300,000);  $c$  = distance from neutral axis where stress is to be found (0 to 10 in.);  $I$  = moment of inertia of section (100 to 10,000 in.<sup>4</sup>).

$$83. \quad P = 0.196 \frac{d^3}{r} f \text{ (load supported by a helical compression spring)}$$

where  $d$  = diameter of wire corresponding to B. & S. gage numbers (0000 to 10);  $r$  = mean radius of spring (0.5 to 2 in.);  $f$  = shearing stress of material (10,000 to 60,000 psi).

$$84. \quad d = 68.5 \sqrt[3]{\frac{H}{\pi(T_{\max})}} \text{ (required diameter of a shaft in torsion)}$$

where  $H$  = horsepower to be transmitted—varies from (0 to 500 hp);  $\pi$  = speed of rotation, rpm (0 to 4000);  $T_{\max}$  = working stress in shear, pounds per square inch (5000 to 15,000);  $d$  = diameter (0 to 3 in.).

$$85. \text{ Euler's column formula: } \frac{P}{A} = \frac{C\pi^2 E}{\left(\frac{L}{\rho}\right)^2}$$

where  $P/A$  = critical average stress in pounds per square inch, varies from (500 to 20,000);  $C$  = fixity coefficient (1 to 4);  $E$  = modulus of elasticity (1 to 30 million psi);  $L/\rho$  = radius of gyration (70 to 200).

### Civil

$$86. \quad X = \frac{l^3}{8R_e L_e} \text{ (offset from the tangent in a spiral easement curve, range 0 to 75)}$$

where  $l$  = distance in feet from T.S. (point of spiral), range (0 to 600);  $R_e$  = radius of circle in feet (300 to 6000);  $L_e$  = total length of spiral in feet (0 to 600).

87.  $S = \frac{l^2}{2R_c L_c}$  (the "spiral angle" or total inclination of curve to tangent at any point on a spiral easement curve, range,  $0^\circ$  to  $30^\circ$ )

where  $l$  = distance in feet from T.S. (point of spiral), range (0 to 600);  $R_c$  = radius of circle in feet (300 to 6000);  $L_c$  = total length of spiral in feet (0 to 600).

88. Since  $D = \frac{5729.65}{R_s}$  (double-graduate the  $R_s$  scale in problem 86 or 87 to read  $D$ , range  $1''$  to  $20''$ )

89.  $e = \frac{gv^2}{32.2R}$  (elevation of track in feet)

where  $g$  = gauge of track;  $v$  = velocity in feet per second (0 to 60);  $R$  = radius of curve in feet (300 to 6000).

90.  $C_s = \frac{W^2 L^3}{24P^3}$  (correction to a steel tape due to sag [0 to 0.5])

where  $W$  = weight of tape in pounds per foot (0 to 0.04);  $L$  = length of tape between supports (0 to 100 ft);  $P$  = applied tension in pounds (0 to 10).

91.  $E = 15[V - e] \left[ 1 + \frac{w}{10} \right]$  (Meyer's evaporation formula)

where  $E$  is the evaporation in inches per 30-day month (0 to 15 in.).

Make the substitution for  $v = (\text{R.H.}) V$ , where  $v$  is the actual vapor pressure, R.H. the relative humidity, and  $V$  the vapor pressure at 100% R.H. and is a function of temperature as given in problem 10, functional scales.  $W$  is the average wind velocity in miles per hour, varying from (0 to 30) and R.H. is (30% to 90%).

### Mechanical

92.  $\text{HP} = \frac{2\pi LNW}{33,000}$  (horsepower as measured by a prony brake [HP, 0 to 75])

where  $L$  = length of brake arm in feet (0.5 to 1.5);  $N$  = shaft speed, revolutions per minute (0 to 4000);  $W$  = load on scales (0 to 200 lb).

93.  $\frac{P_1}{P_2} = \left[ \frac{V_2}{V_1} \right]^{1.41}$  (adiabatic expansion of air)

where  $P_1$  = initial pressure (0 to 300 psi);  $P_2$  = final pressure (0 to 275 psi);  $V_1$  = initial volume (0 to 100 cu ft);  $V_2$  = final volume (0 to 110 cu ft).

94.  $M = 0.3155A_1 \sqrt{\frac{P_1}{V_1}}$  (discharge from a steam nozzle in a turbine in pounds per second [0 to 6])

where  $A_1$  = exit area of nozzle in square inches (0 to 5);  $P_1$  = pressure of steam (15 to 300 psi);  $V_1$  = specific volume, cubic feet per pound (0 to 26).

$$95. \quad W = \frac{D \times H^2 \times F}{B} \quad (\text{minimum weight of square chimney required to withstand force of wind [0 to 300,000]})$$

where  $D$  = average width of side in feet (0 to 10);  $H$  = height of chimney in feet (0 to 100);  $F$  = force of wind = 50 for hurricane which is design condition;  $B$  = breadth of base in feet (0 to 15).

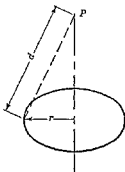
96. Time required for turning or boring work in the lathe is expressed by  $T = L/FN$ , where  $T$  = time in minutes for one cut over the work;  $L$  = length of cut in inches (6 to 72);  $N$  = rpm (10 to 1000);  $F$  = feed in inches per revolution (0.002 to 0.30).

97. Time required for planing and shaping is expressed by  $T = W/FN$ , where  $T$  = time in minutes;  $W$  = width of work in inches (3 to 60);  $F$  = feed per stroke in inches (0.01 to 0.25);  $N$  = number of cutting strokes per minute (2 to 75).

### Electrical

$$98. \quad H = \frac{2\pi I r^2}{d^3} \quad (\text{field intensity at any point } P \text{ as shown by the figure})$$

where  $I$  = current in wire in ab-amperes (0 to 500);  $r$  = in centimeters (1 to 10);  $d$  = in centimeters (1 to 20);  $H$  = in lines per square centimeter (0 to 50).



$$99. \quad R = \frac{\rho}{10^6} \frac{l}{D^2} \quad (\text{resistance of a wire in ohms [0 to 25]})$$

where  $l$  = length of wire in feet (0 to 100);  $\rho$  = specific resistance (0 to 1000);  $D$  = diameter of the wire (0 to 0.1 in.).

$$100. \quad \frac{J}{V} = n_h B_{\max}^{1.6} \quad (\text{ergs}) \quad (\text{hysteresis loss per cubic centimeter per cycle in iron})$$

where  $n_h$  = constant varying from (0.001 to 0.004) for different types of iron;  $J$  = hysteresis loss in ergs (or convert to watts since one erg =  $10^{-7}$  watt)



(0 to 50 watts);  $V$  = volume of iron in cubic centimeters (0 to 1000);  $B_{\max}$  = maximum flux density (0 to 20,000).

$$101. \quad \cos \phi = \frac{P}{EI}$$

where  $\phi$  is the phase angle in an alternating current circuit,  $\phi$  varies ( $0^\circ$  to  $60^\circ$ );  $P$  = power in watts as measured by a wattmeter (0 to 1000);  $E$  = voltage of circuit (0 to 250);  $I$  = current in amperes (0 to 20).

### Aeronautical

$$102. \quad L = 0.00256 C_L A V^2 \text{ (lift of an airfoil in pounds [1000 to 20,000])}$$

where  $C_L$  = lift coefficient of airfoil section (0 to 2.0);  $A$  = area of airfoil, square feet (100 to 1000);  $V$  = velocity in miles per hour (50 to 300)

$$103. \quad P_{pr} = \frac{550n(\text{HP})}{V} \text{ (propeller thrust in pounds [100 to 1000])}$$

where  $n$  = propeller efficiency in % varying from (65 to 90%); HP = engine horsepower (25 to 600);  $V$  = velocity of airplane in feet per second (50 to 400).

$$104. \quad V = 77.3 \left( n \frac{d}{P} \right)^{1/2} \text{ (airspeed at level in feet per second [50 to 400])}$$

where  $n$  = propeller efficiency (65 to 90%);  $d = W/A_D$ , where  $W$  is the weight of the airplane and  $A_D$  is the equivalent drag area in square feet;  $d$  varies from (150 to 1000);  $P = W/\text{HP}$ , where HP is the horsepower of the engine,  $P$  varies from (1 to 15).

$$105. \quad R = \frac{(Kb)^2}{A} \text{ (aspect ratio of a wing, varies [4 to 8])}$$

where  $K$  = Moak's span factor for biplanes, for monoplanes  $K = 1.0$ , varies from (1 to 1.5);  $b$  = span of longest wing in feet (20 to 80),  $A$  = total wing area in square feet (0 to 1500).

### Chemical

$$106. \quad Q = 0.010386 \frac{\alpha}{k} i \text{ (Faraday's law of electrolysis; } Q = \frac{m}{t} \text{ is the quantity deposited per second due to electrolysis and varies [0 to 10])}$$

where  $\alpha$  = the atomic weight (select a number of elements used in electrolysis),  $k$  = the valence of the element (1 to 4);  $i$  = electric current in amperes (1 to 10).

$$107. \quad W = \frac{144mP}{1544(t + 460)} \text{ (weight of a gas [0 to 6 lb per cu ft])}$$

where  $P$  = pressure in pounds per square inch absolute (10 to 1000),  $m$  = molecular weight (2 to 200);  $t$  = temperature ( $0^\circ$  to  $600^\circ\text{F}$ ).

108.  $W = VNM$  (titration equation where  $V$  milliliters of  $N$  normal reagent are required to titrate  $W$  grams of a substance, the milliequivalent of which is  $M$ )

where  $V$  = milliliters (10 to 25);  $N$  = normal reagent (0.1 to 0.5);  $M$  = milliequivalent (0.02 to 0.20);  $W$  = grams of substance (0 to 2.5).

### General

109.  $C = \frac{WV^2}{gR}$  (centrifugal force acting on a body due to a rotation)

where  $W$  = weight in pounds of the body (1 to 150);  $V$  = velocity in feet per second (1 to 50);  $R$  = the radius of the path in feet (0.1 to 10);  $g = 32.2$ ;  $C$  = centrifugal force in pounds (0 to 1500).

110.  $\frac{u}{v} = \frac{20 + t_1}{20 + t_2}$

where  $u$  varies from 0 to 10;  $v$  varies from 0 to 10;  $t_1$  varies from 0 to 100;  $t_2$  varies from 0 to 100.

111.  $(\Delta L) = L \alpha (t_2 - t_1)$ , increase in length of a bar due to temperature changes

where  $L$  = length of bar (0 to 100 ft) and  $\alpha$  = coefficient of expansion as given by the following table:

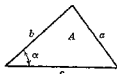
Aluminum	= 0.0000244	Hardened steel	= 0.000010
Lead	= 0.000029	Copper	= 0.0000171
Wrought iron and mild steel	= 0.000011	Brass	= 0.0000198
Crown glass	= 0.000009	Tin	= 0.000027

$(t_2 - t_1)$ , change in temperature in degrees Centigrade ( $0^\circ$  to  $100^\circ$ )

112.  $I = PRT$ , (simple interest law)

where  $I$  = interest (0 to \$400);  $P$  = principal (\$1 to \$1000);  $R$  = rate of interest per year period ( $\frac{1}{4}$  to 8%);  $T$  = time or period in years (0 to 5 years) (subdivide time scale into months).

113.  $A = \frac{1}{2}bc \sin \alpha$  = area of a triangle shown in the figure



where  $\alpha = (20^\circ$  to  $160^\circ)$ ;  $b = (0$  to  $10)$ ;  $c = (0$  to  $10)$ ;  $A = (0$  to  $50)$ .

## Chapter Nine

### PROPORTIONAL CHARTS OF THE FORM

$$f_1(u) + f_2(v) = \frac{f_3(w)}{f_4(q)}$$

An equation of the above form can be solved by a combination of two types already discussed (Figure 64).

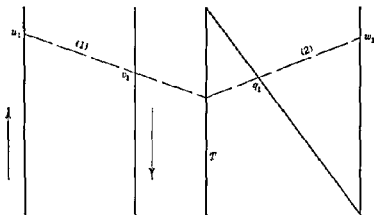


FIG. 64.

Let  $f_1(u) + f_2(v) = T$  (1) (3 parallel scales)  
 and  $T = \frac{f_3(w)}{f_4(q)}$  (2) (Z chart)  
 or  $f_3(w) = Tf_4(q)$

Now let us consider another method for solving the above equation (Figure 65). Suppose that the parallel scales, *A* and *B*, are graduated in accordance with the scale equations:

$$X_u = m_u f_1(u)$$

$$X_v = m_v f_2(v)$$

and that the scale *A* also carries graduations for the  $f_3(w)$ , the scale equation of which is  $X_w = m_w f_3(w)$ . Let us further suppose that the

- (a) Graduate the left side of scale  $A$  from its scale equation,  $X_u = m_u f_1(u)$ .

- (b) Graduate scale  $B$  from  $X_v = m_v f_2(v)$ .  
 (c) Graduate the right side of scale  $A$  from its scale equation,  $X_w = m_w f_3(w)$ .  
 (d) Graduate the diagonal scale from  $X_q = m_q f_4(q)$ , where  $m_q = Km_w/m_v$ .

*Caution:* Do not overlook the fact that point  $C$  is the zero value of functions  $v$ ,  $w$ , and  $q$ ; that point  $D$  is the zero value of function  $v$ .

### EXAMPLE

$$V = \frac{\pi h}{9} \left( \frac{5}{4} D^2 + d^2 \right) \text{ (volume of a buoy) (Figure 66)}$$

where  $h$  = height of buoy (0 to 10 ft);  $D$  = diameter of midsection (0 to 10 ft),  $d$  = diameter of base (0 to 10 ft); and  $V$  = volume of buoy.

Then  $\frac{5}{4} D^2 + d^2 = \frac{9V}{\pi h}$ , which is of the form

$$f_1(u) + f_2(v) = \frac{f_3(w)}{f_4(q)}$$

$$X_D = m_D \frac{5}{4} D^2 = 0.06 \frac{5}{4} D^2 = 0.075 D^2 \text{ (scale length, 7.5 in.)}$$

$$X_d = m_d d^2 = 0.06 d^2 \text{ (scale length, 6 in.)}$$

$$X_v = m_v 9V = 0.001 \times 9V = 0.009V \text{ (scale length approx. 7 in.)}$$

$$\frac{m_v}{m_h} = \frac{m_D}{K}$$

$$\begin{aligned} \text{or } m_h &= \frac{Km_v}{m_D} = \frac{0.001 \times K}{0.06} = \frac{0.130}{6\pi} \\ &= \frac{0.5}{\pi}, \text{ when } K = \frac{30}{\pi} = 9.55 \text{ in.} \end{aligned}$$

$$X_h = m_h \pi h = \frac{0.5}{\pi} \pi h = 0.5h$$

**Example:** *Given:*  $D = 8$ ,  $d = 10$ ;  $h = 5$ . *Solution:* Join points 8 and 10 on the  $D$  and  $d$  scale, respectively. Through point 5 on the  $h$  scale draw a parallel line. This line cuts the  $V$  scale in point  $V = 315$ . By computation,  $V = 314.20$ .

If the equation is of the form  $f_1(u) - f_2(v) = f_3(w)/f_4(q)$ , positive values of  $u$  and  $v$  will be laid off in the same direction. This is shown in Figure 67.

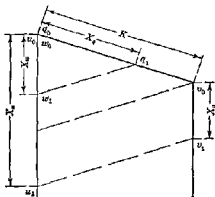


FIG. 67.

$$\frac{X_u - X_v}{K} = \frac{X_w}{X_q}$$

Again, if

$$X_u = m_u f_1(u)$$

$$X_v = m_v f_2(v)$$

$$X_w = m_w f_3(w)$$

$$X_q = m_q f_4(q)$$

and

$$\frac{m_u}{K} = \frac{m_w}{m_q}$$

Then

$$f_1(u) - f_2(v) = \frac{f_3(w)}{f_4(q)}$$

#### EXAMPLE

$$u^2 - v^2 = \frac{w}{4q} \quad (\text{Figure 68})$$

$u$  (0 to 5);  $v$  (0 to 5);  $q$  (0 to 6)

**Example:** Given:  $u = 5$ ;  $v = 3$ ;  $q = 4$ .

**Required:**  $w$ .

**Solution.** Join points 5 and 3 on the  $u$  and  $v$  scales, respectively. Through point 4 on the  $q$  scale, draw a line parallel to line 1 and read  $w = 255$ . By computation,  $w = 256$ .

If it is desired to construct a nomogram for an equation of the type form  $f_1(u) + f_2(v) = f_3(w)/f_4(q)$  so that no double scales will be neces-

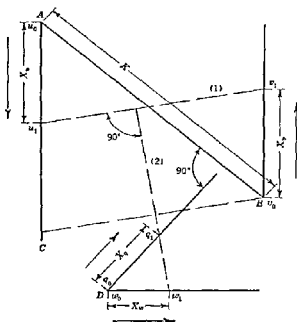


FIG. 69.

sary, another arrangement can be made which will overcome this situation.

Let us consider Figure 69. Scales  $u$  and  $w$  are at right angles. Similarly, scale  $q$  and the diagonal  $AB$ , which joins the zero values of functions  $u$  and  $v$ , are at right angles. Scales  $u$  and  $v$  are parallel.

The geometric relations can easily be determined by studying a typical case. Suppose line 1 joins any two points  $u_1$  and  $v_1$ . Then let us

draw line 2 through point  $w_1$  and perpendicular to line 1. The intersection of line 2 with the  $q$  scale will give us point  $q_1$ , the desired solution. Why is this true?

In Figure 69 it will be seen that line  $BC$  is parallel to line 1. Again, triangles  $ABC$  and  $Dq_1w_1$  are similar. Therefore

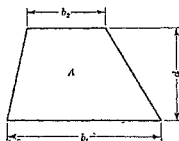
$$\frac{X_u + X_v}{K} = \frac{X_w}{X_q}$$

The remainder of the development is the same as shown previously.

There is an advantage in this design over the one which uses parallels, in that (1) there is a separate scale for each function and (2) the readings can be made by placing a transparent sheet, having but two lines at right angles, over the nomogram. Proper orientation of the lines can be made very quickly.

### EXERCISES

114.  $A = \frac{d(b_1 + b_2)}{2}$  (area of a trapezoid)



where  $b_1 = (0 \text{ to } 100)$ ;  $b_2 = (0 \text{ to } 80)$ ;  $d = (0 \text{ to } 50)$ ;  $A =$  limits corresponding.

115. Weight of a hollow steel tube:  $W = \frac{\ell\pi(d^2 - d_1^2)}{4} \cdot \rho$

where  $W =$  the weight (0 to 100 lb);  $\ell =$  the length in inches (0 to 100);  $d =$  the outside diameter (0 to 2 in.);  $d_1 =$  the inside diameter (0 to 1.9 in.);  $\rho =$  density = 480.6/1728 lb per cu in.

116.  $KV = (p_1 + p_2)$

where  $V =$  the volume of earthwork per station;  $p_1 + p_2 =$  average planimeter readings in square inches from the cross section drawings, and  $K =$  a constant depending on the length of section and the scale;  $V = (0 \text{ to } 1000)$  in cubic yards;  $p_1$  or  $p_2 = (0 \text{ to } 10)$ , and  $K =$  corresponding limits.



$$117. \quad F = \frac{k}{t^a} \quad (\text{Wall Thickness Sensitivity})$$

where  $F$  is the tensile strength of a metal or alloy,  $t$  is the thickness,  $k$  and " $a$ " are constants depending on the kind of material.

$$F = (10 \text{ to } 50 \text{ kg per sq mm})$$

$$k = \text{constant (10 to 50 kg per sq mm)}$$

$$t = (10 \text{ to } 90 \text{ mm})$$

$$a = (0.2 \text{ to } 0.7)$$

$$118. \quad R = (P - d)55,000 \quad (\text{strength of a riveted steel plate between rivet holes})$$

where  $P$  is the pitch of the rivets and  $d$  the diameter.

$$P = (0 \text{ to } 5 \text{ in.})$$

$$d = (0 \text{ to } 1 \text{ in.})$$

$$R = (0 \text{ to } 100,000 \text{ lb})$$

$$t = (0 \text{ to } 1 \text{ in.})$$

$$119. \quad \frac{T_d}{T_s} = e^{\mu\theta} \quad (\text{ratio of tensions in a rope used over a pulley as shown in the figure where } T_d \text{ is the larger pull})$$

$$T_d \text{ varies (1 to 100 lb)}$$

$$T_s \text{ varies (1 to 100 lb)}$$

$$\mu \text{ varies (0 to 0.4)}$$

$$\theta \text{ varies (0 to } \pi)$$



## Chapter Ten

### ALIGNMENT CHARTS FOR THE SOLUTION OF EQUATIONS OF THE FORM

$$f_1(u) + f_2(v) \cdot f_3(w) = f_4(w)$$

Suppose the parallel scales,  $A$  and  $B$  (Figure 70), are graduated in accordance with the scale equations:

$$X_u = m_u f_1(u)$$

$$X_v = m_v f_2(v)$$

It is further supposed that a straight line joining points  $u_1$  and  $v_1$  cuts the curved scale,  $C$ , in point  $w_1$ , which satisfies the equation.

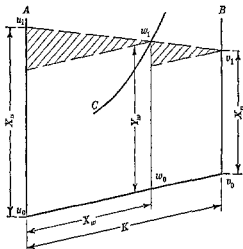


FIG. 70.

Points on the scale  $C$  are located by co-ordinates,  $X_w$  and  $Y_w$ . Let us develop expressions for  $X_w$  and  $Y_w$ .

From the similar triangles (shaded),

$$\frac{X_u - Y_w}{Y_w - X_v} = \frac{X_w}{K - X_w}$$

from which  $X_u(K - X_w) + X_v X_w = KY_w$

or  $X_u + X_v \left[ \frac{X_w}{K - X_w} \right] = \left[ \frac{KY_w}{K - X_w} \right]$

Now  $m_u f_1(u) + m_v f_2(v) \left[ \frac{X_w}{K - X_w} \right] = \left[ \frac{KY_w}{K - X_w} \right]$

This is true since  $X_u = m_u f_1(u)$  and  $X_v = m_v f_2(v)$ . Careful study of the above equation will show that  $X_w/(K - X_w)$  must equal  $Cf_3(w)$  and that the right-hand member,  $KY_w/(K - X_w)$  must equal  $C_1 f_4(w)$ . In order to obtain the equation  $f_1(u) + f_2(v) f_3(w) = f_4(w)$ , it will be seen that  $C = m_u/m_v$  and  $C_1 = m_u$ . This means that

$$\frac{X_w}{K - X_w} = \frac{m_u}{m_v} f_3(w) \quad \text{and} \quad \frac{KY_w}{K - X_w} = m_u f_4(w)$$

From  $\frac{X_w}{K - X_w} = \frac{m_u}{m_v} f_3(w)$

$$X_w = \frac{Km_u f_3(w)}{m_u f_3(w) + m_v}$$

and from  $\frac{KY_w}{K - X_w} = m_u f_4(w)$

$$Y_w = \frac{Km_u f_4(w) - X_w m_u f_4(w)}{K}$$

or  $Y_w = m_u f_4(w) - \frac{m_u f_3(w) m_u f_4(w)}{m_u f_3(w) + m_v}$

$$Y_w = \frac{m_u m_v f_4(w)}{m_u f_3(w) + m_v}$$

Hence, to construct an alignment chart of the above form,

(1) Graduate the *A* and *B* scales from their scale equations:

$$X_u = m_u f_1(u)$$

$$X_v = m_v f_2(v)$$

(2) Locate the curved scale by its co-ordinates:

$$X_w = \frac{Km_u f_3(w)}{m_u f_3(w) + m_v}$$

$$Y_w = \frac{m_u m_v f_4(w)}{m_u f_3(w) + m_v}$$

*Caution.* The axis from which distances  $Y_w$  are laid off is the line which joins the zero value of function,  $f_1(u)$ , with the zero value of function,  $f_2(v)$ .

### EXAMPLE

$$w^2 + pw + q = 0 \text{ (quadratic formula)}$$

Transposing  $q + pw = -w^2$

which is of the form

$$f_1(u) + f_2(v) \cdot f_3(w) = f_4(w)$$

$$X_q = m_q q = 0.6q$$

$$X_p = m_p p = 0.6p$$

where  $m_q$  and  $m_p$  were arbitrarily chosen as 0.6.

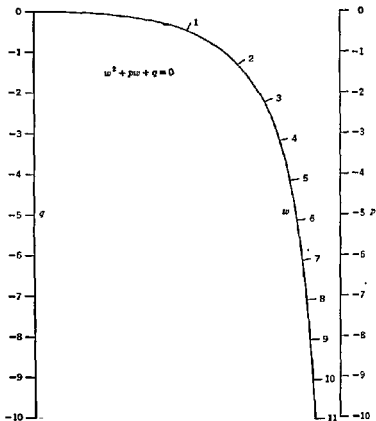
$$X_w = \frac{Km_u f_3(w)}{m_u f_3(w) + m_v}$$

If  $K = 5$  in.

$$X_w = \frac{5 \times 0.6 \times w}{0.6w + 0.6} = \frac{5w}{w + 1}$$

$$\begin{aligned} Y_w &= \frac{m_u m_v f_4(w)}{m_u f_3(w) + m_v} \\ &= \frac{0.6 \times 0.6(-w^2)}{0.6w + 0.6} = \frac{-0.6w^2}{w + 1} \end{aligned}$$

See Figure 71 for graphical solution.

FIG. 71. Alignment Chart for the Equation,  $w^2 + pw + q = 0$ .

## EXERCISES

120.  $Q = 3.33(B - 0.2H)H^{3/2}$  (Francis formula for the quantity of water flowing over a contracted weir)

where  $B$  = width of the weir in feet (0 to 5);  $H$  = head over the crest (0 to 5 ft);  $Q$  = calculate limits.

121.  $S = V_0 t - \frac{1}{2}gt^2$  (distance traveled by a body projected upward with a velocity  $v_0$ , after a time,  $t$ )

where  $S$  = distance in feet (calculate limits);  $v_0$  = velocity in feet per second (0 to 100);  $t$  = time in seconds (0 to 5); and  $g = 32.2$ .

122.  $\left(\frac{I}{y}\right) = 0.0982 \left(\frac{D^4 - d^4}{D}\right)$  (section modulus of a hollow tube whose outside and inside diameters are  $D$  and  $d$ , respectively)

where  $D$  varies (0 to 10 in.) and  $d$  varies (0 to 9 in.).

123.  $V = 0.649 \frac{T}{p} - \frac{22.58}{p^{3/4}}$  (specific volume in cubic feet of superheated steam under a pressure of  $p$  pounds per square inch and with a temperature,  $T$ , in degrees Fahrenheit)

where  $T$  varies (220 to 600);  $p$  = (30 to 200).

124.  $V = \frac{2}{3}\pi r^2 h + \frac{1}{6}\pi h^3$  (volume of a spherical segment with one base)

where  $h$  is the altitude of the segment and  $r$  the radius of the sphere.  $r$  varies (0 to 10 in.) and  $h$  varies (0 to 10 in.).

# Chapter Eleven

## MISCELLANEOUS FORMS

$$1. \frac{f_1(u) + f_2(v)}{f_1(u) - f_2(v)} = \frac{f_3(w)}{f_4(q)} \text{ (Figure 72)}$$

$$X_u = m_u f_1(u)$$

$$X_w = m_w f_3(w)$$

$$X_v = m_v f_2(v)$$

$$X_q = m_q f_4(q)$$

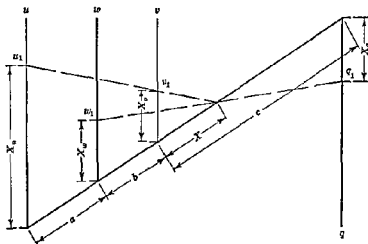


FIG. 72. Chart for an Equation of the Form,  $\frac{f_1(u) + f_2(v)}{f_1(u) - f_2(v)} = \frac{f_3(w)}{f_4(q)}$ .

From similar triangles,

$$\frac{X_u}{X_v} = \frac{a + b + X}{X}$$

$$\frac{X_w}{X_q} = \frac{b + X}{c - X}$$

Eliminating  $X$ ,

$$\frac{X_w}{X_g} = \frac{b + \frac{a+b}{\frac{X_u}{X_v} - 1}}{c - \frac{a+b}{\frac{X_u}{X_v} - 1}}$$

and simplifying,

$$\frac{X_w}{X_g} = \frac{bX_u + aX_v}{cX_u - (a+b+c)X_v}$$

then 
$$\frac{m_w f_3(w)}{m_g f_4(g)} = \frac{bm_u f_1(u) + am_v f_2(v)}{cm_u f_1(u) - (a+b+c)m_v f_2(v)}$$

If 
$$b = \frac{m_v}{m_u} \cdot a, \quad a+b+c = c \frac{m_u}{m_v}$$

and 
$$\frac{m_w}{m_g} = \frac{am_v}{cm_u}$$

it can be shown that

$$\frac{f_1(u) + f_2(v)}{f_1(u) - f_2(v)} = \frac{f_3(w)}{f_4(g)}$$

By algebraic manipulation of the three previous substitutions,

$$\frac{m_u + m_v}{m_u - m_v} = \frac{m_g}{m_w}$$

$$\frac{a}{b} = \frac{m_u}{m_v}$$

$$\frac{a+b+c}{c} = \frac{m_u}{m_v} \quad (\text{Intermediate steps are left to the reader})$$



2. The type form discussed above can be represented by an alignment chart of the design shown in Figure 73. The scales for  $f_1(u)$  and  $f_2(v)$  are at right angles; likewise, the scales for  $f_3(w)$  and  $f_4(q)$  are at right angles; and a  $45^\circ$  angle exists between the  $f_1(u)$  and  $f_3(w)$  scales. If values  $u_1$ ,  $v_1$ , and  $w_1$  are selected, the value of  $q_1$  is obtained by drawing a line through  $w_1$ , parallel to the line joining  $u_1$  with  $v_1$ .

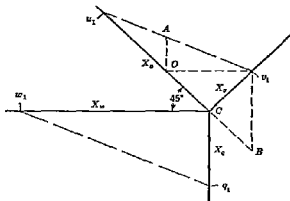


FIG. 73 Alternate Form of Chart for the Equation in Fig. 72.

Let us examine the geometry of the figure. Triangles  $u_1v_1B$  and  $u_1AO$  are similar (from the construction shown). Therefore,

$$\frac{u_1C + CB}{u_1O} = \frac{v_1B}{AO}$$

$$\text{or} \quad \frac{X_u + X_v}{X_u - X_v} = \frac{v_1B}{AO}$$

Also, triangles  $AOv_1$  and  $w_1Cq_1$  are similar. Hence,

$$\frac{Ov_1}{AO} = \frac{X_w}{X_q}$$

$$\text{Since} \quad Ov_1 = v_1B$$

$$\frac{X_u + X_v}{X_u - X_v} = \frac{X_w}{X_q}$$

If  $X_u = m_u f_1(u)$

$$X_v = m_v f_2(v)$$

$$X_w = m_w f_3(w)$$

$$X_q = m_q f_4(q)$$

then 
$$\frac{m_u f_1(u) + m_v f_2(v)}{m_u f_1(u) - m_v f_2(v)} = \frac{m_w f_3(w)}{m_q f_4(q)}$$

This means that  $m_u = m_v$  and  $m_w = m_q$  if

$$\frac{f_1(u) + f_2(v)}{f_1(u) - f_2(v)} = \frac{f_3(w)}{f_4(q)}$$

$$3. \quad f_1(u) \cdot f_2(v) + f_3(w) \cdot f_4(q) = f_5(q)$$

Let  $f_1(u) \cdot f_2(v) = T$  (1)

and  $T + f_3(w) \cdot f_4(q) = f_5(q)$  (2)

Thus a combination of a *Z* chart and one involving two straight lines and a curve can be formed to solve the above equation (Figure 74).

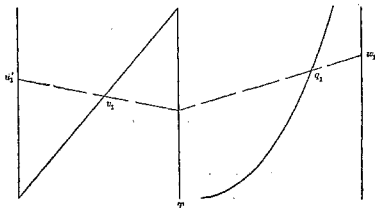


FIG. 74. Alignment Chart for an Equation of the Form,  $f_1(u) \cdot f_2(v) + f_3(w) \cdot f_4(q) = f_5(q)$ .

2. The type form discussed above can be represented by an alignment chart of the design shown in Figure 73. The scales for  $f_1(u)$  and  $f_2(v)$  are at right angles; likewise, the scales for  $f_3(w)$  and  $f_4(q)$  are at right angles; and a  $45^\circ$  angle exists between the  $f_1(u)$  and  $f_3(w)$  scales. If values  $u_1$ ,  $v_1$ , and  $w_1$  are selected, the value of  $q_1$  is obtained by drawing a line through  $w_1$ , parallel to the line joining  $u_1$  with  $v_1$ .

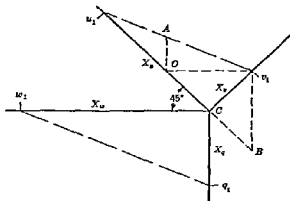


FIG. 73. Alternate Form of Chart for the Equation in Fig. 72.

Let us examine the geometry of the figure. Triangles  $u_1v_1B$  and  $u_1AO$  are similar (from the construction shown). Therefore,

$$\frac{u_1C + CB}{u_1O} = \frac{v_1B}{AO}$$

or

$$\frac{X_u + X_v}{X_u - X_v} = \frac{v_1B}{AO}$$

Also, triangles  $AOv_1$  and  $w_1Cq_1$  are similar. Hence,

$$\frac{Ov_1}{AO} = \frac{X_w}{X_q}$$

Since

$$Ov_1 = v_1B$$

$$\frac{X_u + X_v}{X_u - X_v} = \frac{X_w}{X_q}$$

$$\begin{aligned} \text{If} \quad X_u &= m_u f_1(u) \\ X_v &= m_v f_2(v) \\ X_w &= m_w f_3(w) \\ X_q &= m_q f_4(q) \end{aligned}$$

$$\text{then} \quad \frac{m_u f_1(u) + m_v f_2(v)}{m_u f_1(u) - m_v f_2(v)} = \frac{m_w f_3(w)}{m_q f_4(q)}$$

This means that  $m_u = m_v$  and  $m_w = m_q$  if

$$\frac{f_1(u) + f_2(v)}{f_1(u) - f_2(v)} = \frac{f_3(w)}{f_4(q)}$$

$$3. \quad f_1(u) \cdot f_2(v) + f_3(w) \cdot f_4(q) = f_5(q)$$

$$\text{Let} \quad f_1(u) \cdot f_2(v) = T \quad (1)$$

$$\text{and} \quad T + f_3(w) \cdot f_4(q) = f_5(q) \quad (2)$$

Thus a combination of a Z chart and one involving two straight lines and a curve can be formed to solve the above equation (Figure 74).

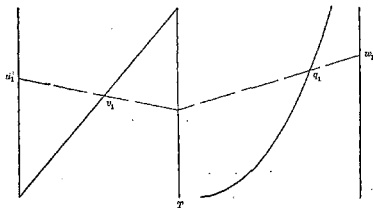


FIG. 74. Alignment Chart for an Equation of the Form,  $f_1(u) \cdot f_2(v) + f_3(w) \cdot f_4(q) = f_5(q)$ .

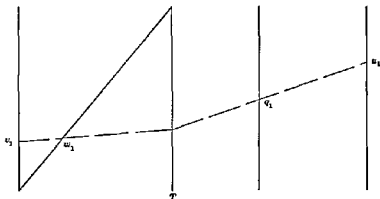
$$4. \quad f_1(u) + f_2(v) \cdot f_3(w) = f_4(q) \quad (\text{Figure 75})$$

Let

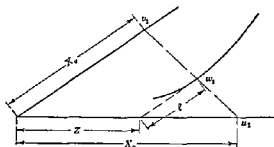
$$f_2(v) \cdot f_3(w) = T$$

and

$$f_1(u) + T = f_4(q)$$

FIG. 75. Alignment Chart for an Equation of the Form,  $f_1(u) + f_2(v) \cdot f_3(w) = f_4(q)$ 

$$5. \quad \frac{1}{f_1(u)} + \frac{f_4(w)}{f_2(v)} = \frac{1}{f_3(w)} \quad (\text{Figure 76})$$

FIG. 76. Alignment Chart for an Equation of the Form,  $\frac{1}{f_1(u)} + \frac{f_4(w)}{f_2(v)} = \frac{1}{f_3(w)}$ .

$$X_u = m_u f_1(u)$$

$$Z = m_u f_3(w)$$

$$X_v = m_v f_2(v)$$

$$l = m_v f_3(w) f_4(w)$$

Points on the curve are located from

$$Z = m_u f_3(w)$$

and

$$l = m_v f_3(w) f_4(w)$$

in the following manner:

1. Graduate a *temporary*  $w$  scale along the horizontal scale for  $f_1(u)$ .
2. Draw lines through points on the temporary  $w$  scale, parallel to the  $v$  scale.
3. On these lines lay off distances obtained from  $l = m_v f_3(w) f_4(w)$ , using the same value of  $w$  through which the parallels were drawn.

$$6. \quad f_1(u) \cdot f_2(v) \cdot f_3(w) = f_4(q) \cdot f_5(r) \quad (\text{Figure 77})$$

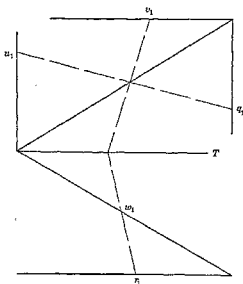


FIG. 77. Combination Proportional and  $Z$  Chart for an Equation of the Form,  
 $f_1(u) \cdot f_2(v) \cdot f_3(w) = f_4(q) \cdot f_5(r)$ .

Let

$$\frac{f_1(u)}{f_4(q)} = \frac{T}{f_2(v)} \quad (\text{proportional chart}) \quad (1)$$



## 3. DESIGN OF NET CHARTS

Problems involving four variables may also be solved by a net chart which makes it possible to read all four variables with one isopleth. The principles involved in the design of this type chart are the same as those employed in the design of a chart of the form,  $f_1(u) + f_2(v) = f_3(w)$ .

## EXAMPLE

Suppose the given equation is  $S = V_0 t + \frac{1}{2} a t^2$ , where  $S$  = distance traversed in feet (0 to 15),  $V_0$  = initial velocity in feet per second (0 to 10),  $a$  = acceleration in feet per second<sup>2</sup> (0 to 4), and  $t$  = time interval in seconds (1 to 4).

*Solution.* Let  $t = 1, 2, 3$ , and  $4$ . With these values for  $t$ , the following equations result, namely:

$$S = V_0 + \frac{a}{2} \quad (1)$$

$$S = 2V_0 + 2a \quad (2)$$

$$S = 3V_0 + \frac{9}{2}a \quad (3)$$

$$S = 4V_0 + 8a \quad (4)$$

All the above equations are of the form  $f_1(u) + f_2(v) = f_3(w)$ .

Consider the first equation,  $S = V_0 + a/2$ . It may be written  $V_0 - S = -a/2$  to conform with the type equation  $f_1(u) + f_2(v) = f_3(w)$ . Suppose that the desired length of the  $V_0$  and  $S$  scales is 10 units; then, the scale equations are:

$$X_{v_0} = V_0 \quad \text{and} \quad X_s = \frac{2}{3}S.$$

From the above moduli, the modulus for the  $a$  scale is  $\frac{1 \times \frac{2}{3}}{1 + \frac{2}{3}} = \frac{2}{5}$ ; and

its position is determined from the ratio  $\frac{1}{\frac{2}{3}} = \frac{3}{2}$ . The scale equation for  $a$  then becomes  $X_a = \frac{3}{2}(a/2) = a/5$ . The chart for the equation  $V_0 - S = -a/2$  is shown in Figure 79.

Now, consider equation 2,  $S = 2V_0 + 2a$ . This equation can be rewritten in type form as  $2V_0 - S = -2a$ .

If we are to use the same  $V_0$  and  $S$  scales as shown in Figure 79, the effective moduli for the scale equations,  $X_{v_0} = m_{v_0}(2V_0)$  and  $X_s = m_s S$ , must be the same as those used in equation 1.

This means that  $m_s$  must equal  $\frac{1}{2}$  in order for the effective modulus



to equal one. Therefore, the scale equation for  $V_0$  in equation 2 is  $X_{v_0} = \frac{1}{2}(2V_0) = V_0$ .

Since the coefficient of  $S$  is the same in both equations 1 and 2, no change in modulus for the  $S$  scale is necessary. Note, however, that the location of the  $\alpha$  scale is determined by the ratio  $\frac{m_{v_0}}{m_s} = \frac{\frac{1}{2}}{\frac{1}{2}} = \frac{3}{4}$ .

The chart for equation 2 is shown in Figure 80.

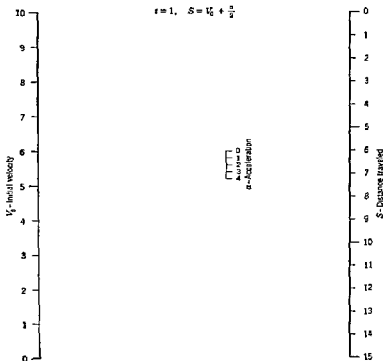


FIG. 79. First Step in Designing a Net Chart for the Equation,  $S = V_0 t + \frac{1}{2} a t^2$ .

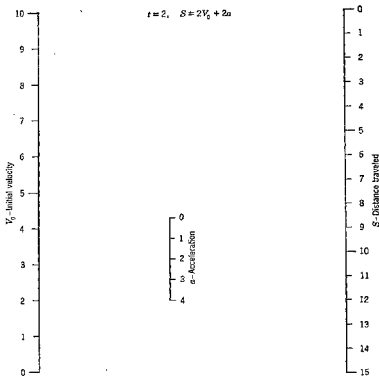


FIG. 80. Second Step in Designing a Net Chart for the Equation,  $S = V_0 t + \frac{1}{2} a t^2$ .

If the two charts are superposed, the resulting chart, Figure 81, would be obtained.

It should be clear that similar calculations for equations 3 and 4 would be necessary to complete the net chart. It is not necessary to

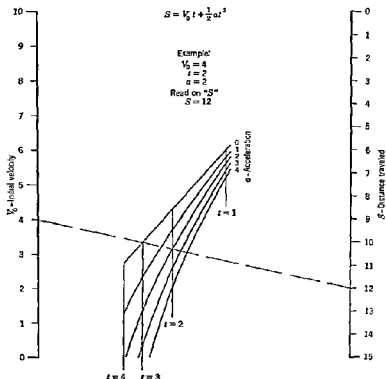


FIG. 81. Net Chart for the Equation,  $S = V_0 t + \frac{1}{2} a t^2$ .

make separate charts for each of the four equations, since the  $V_0$  and  $S$  scales are the same in all cases. Calculations for positioning the  $a$  scale are necessary and, in addition, the moduli for the  $a$  scales must be computed in order to graduate these scales properly. Finally, curves drawn through like values of  $a$  will establish the net for the variable  $a$ . The net for the variable,  $t$ , consists of the vertical lines which first carried the values of  $a$  when  $t$  equaled 1, 2, 3, and 4. The completed chart is shown in Figure 81.

## EXERCISES

125.

$$f = \frac{9Wl}{bd^2}$$

when  $f$  = fiber stress (500 to 2500 psi);  $l$  = length of wooden beam (5 to 30 ft);  $W$  = total load (1000 to 12,000 lb);  $b$  = width of beam (2 to 12 in.);  $d$  = depth of beam (4 to 16 in.).

126.

$$\Delta_{\max} = \frac{Pl^3}{48EI}$$

where  $\Delta$  = deflection of a simple beam with a concentrated load at the center, in inches;  $P$  = concentrated load (500 to 10,000 lb);  $l$  = length of beam (60 to 300 in.);  $E$  = modulus of elasticity ( $2 \times 10^6$  to  $30 \times 10^6$  psi);  $I$  = moment of inertia (1000 to 20,000 in.<sup>4</sup>).

127.

$$\Delta_{\max} = \frac{Wl^3}{KEI}$$

where  $\Delta$  is deflection in inches of simple beams loaded as follows: (a) uniformly ( $K = 384/5$ ), (b) load increasing uniformly to one end ( $K = 1000/13$ ), (c) load increasing uniformly to the center ( $K = 60$ );  $W$  = total load (10,000 to 300,000 lb);  $E$  and  $I$  as above; and  $l$  = (120 to 600 in.).

## SUMMARY OF TYPE FORMS

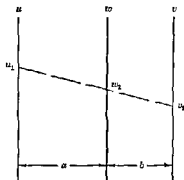
$$1. \quad f_1(u) + f_2(v) = f_3(w)$$

Scale equations:

$$X_u = m_u f_1(u)$$

$$X_v = m_v f_2(v)$$

$$X_w = \frac{m_u m_v}{m_u + m_v} f_3(w)$$



$$f_1(u) + f_2(v) = f_3(w)$$

Scale location:

$$\frac{a}{b} = \frac{m_u}{m_v}$$

$$2. \quad f_1(u) = f_2(v) \cdot f_3(w)$$

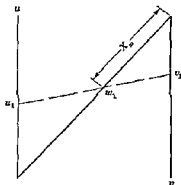
Scale equations:

$$X_u = m_u f_1(u)$$

$$X_v = m_v f_2(v)$$

$$X_w = \frac{K}{K_1 f_3(w) + 1}$$

where  $K_1 = \frac{m_u}{m_v}$  and  $K$  is the length of the diagonal.



$$f_1(u) = f_2(v) \cdot f_3(w)$$

3.

$$f_1(u) + f_2(v) = \frac{f_1(u)}{f_3(w)}$$

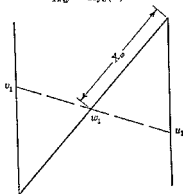
Scale equations:

$$X_u = m_u f_1(u)$$

$$X_v = m_v f_2(v)$$

$$m_u = m_v$$

$$X_w = K f_3(w)$$

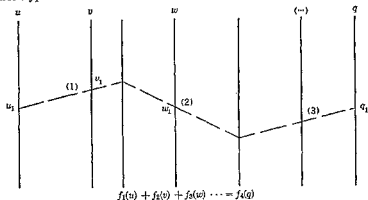


$$f_1(u) + f_2(v) = \frac{f_1(u)}{f_3(w)}$$



$$5. \quad f_1(u) + f_2(v) + f_3(w) \cdots = f_4(q)$$

See Type 1.



6.

$$\frac{f_1(u)}{f_2(v)} = \frac{f_3(w)}{f_4(q)}$$

Scale equations:

$$X_u = m_u f_1(u)$$

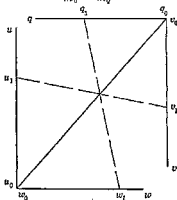
$$X_w = m_w f_3(w)$$

$$X_v = m_v f_2(v)$$

$$X_q = m_q f_4(q)$$

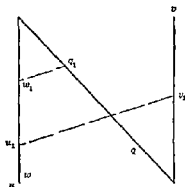
and

$$\frac{m_u}{m_v} = \frac{m_w}{m_q}$$



$$\frac{f_1(u)}{f_2(v)} = \frac{f_3(w)}{f_4(q)}$$





$$f_1(u) + f_2(v) = \frac{f_3(w)}{f_4(q)}$$

$$7. \quad f_1(u) + f_2(v) = \frac{f_3(w)}{f_4(q)}$$

Scale equations:

$$X_u = m_u f_1(u)$$

$$X_v = m_v f_2(v)$$

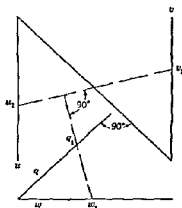
$$X_w = m_w f_3(w)$$

$$m_u = m_v$$

$$X_q = m_q f_4(q) \text{ where } m_q = \frac{K m_w}{m_u}$$

$$8. \quad f_1(u) + f_2(v) = \frac{f_3(w)}{f_4(q)}$$

Same as in 7. Alternate form of chart shown below.



$$\text{Alternate Form for } f_1(u) + f_2(v) = \frac{f_3(w)}{f_4(q)}$$

9.

$$f_1(u) + f_2(v)f_3(w) = f_4(w)$$

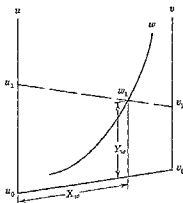
Scale equations:

$$X_u = m_u f_1(u)$$

$$X_v = m_v f_2(v)$$

$$X_w = \frac{Km_u f_3(w)}{m_u f_3(w) + m_v}$$

$$Y_w = \frac{m_v m_v f_4(w)}{m_u f_3(w) + m_v}$$



$$f_1(u) + f_2(v) \cdot f_3(w) = f_4(w)$$

## Chapter Twelve

### PRACTICAL SHORT-CUTS IN THE DESIGN OF ALIGNMENT CHARTS

If the designer is thoroughly grounded in the theory of alignment charts and fully understands the mathematical methods employed in changing a given equation to a type form, it is frequently possible to short-cut the actual construction of the chart.

#### EXAMPLE 1

Suppose that the given equation is  $M = w^2 l / 8$  (bending moment in foot-pounds), where the ranges are  $w$  (10 to 300 lb per ft) and  $l$  (5 to 30 ft).

If a chart consisting of parallel scales is desired, the designer recognizes the fact that the equation can be converted to the form:

$$\log w + 2 \log l = \log M + \log 8$$

The chart can now be constructed without making any further calculations. The following procedure is suggested:

1. Draw two parallel lines any convenient distance apart.
2. Graduate the left-hand scale for  $w$  by simply marking the lower point 10 and the upper point 300. Other points on the scale may be located by projecting from a log scale (two-deck slide rule scale or commercial log sheets having two decks).
3. Mark the lower point of the  $l$  scale 5 and the upper point 30. Again, locate additional graduations by projecting from a log scale.
4. Now calculate two points for  $M$ , i.e.:  
(a) Let  $w = 40$  and  $l = 10$ . This yields  $M = 500$ .  
(b) Let  $w = 160$  and  $l = 5$ . This yields  $M = 500$ .

The point in which the line joining 40 and 10 intersects the line joining 160 and 5 is point  $M = 500$ . The vertical line through this point locates the  $M$  scale. A second point on this scale can be now located

by letting  $w = 40$  and  $l = 30$ , which yield  $M = 4500$ . The line joining  $w = 40$  with  $l = 30$ , then cuts the  $M$  scale in point 4500. Other points may be obtained by projecting from a log scale. The completed chart is shown in Figure 82.

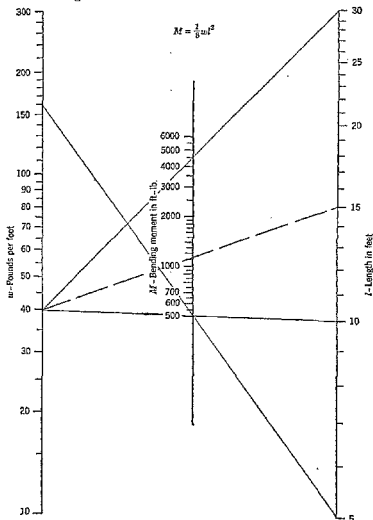


FIG. 82. Alignment Chart for the Equation,  $M = \frac{1}{8} w l^2$ , Constructed by the Short-Cut Method.

## EXAMPLE 2

Consider the equation,

$$S = 0.0982 \left( \frac{D^4 - d^4}{D} \right) \text{ (section modulus for tubes and bars)}$$

where  $D$  and  $d = (0 \text{ to } 10 \text{ in.})$  and  $S = (0 \text{ to } 100 \text{ in.}^3)$ .

The equation may be converted to the form,  $0.0982d^4 + DS = 0.0982D^4$ , which is in the type form,  $f_1(u) + f_2(v) \cdot f_3(w) = f_4(w)$ .

The chart will consist of two parallel scales for  $d$  and  $S$  respectively, and a curved scale for  $D$ . A preliminary sketch of the chart would look something like Figure 83.

The following procedure is suggested:

1. Draw the  $d$  and  $S$  scales a convenient distance apart.

2. Mark the lower point on the  $d$  scale, 0 and the upper point 10.

3. Likewise, mark the lower and upper points on the  $S$  scale 0 and 100, respectively. Since

the function of  $S$  is linear, this scale will be uniform and can be readily graduated.

4. The  $d$  scale can be graduated by first laying out a  $d^4$  scale and then projecting this scale to the  $d$  scale.

5. If  $S = 0$ , then  $d^4 = D^4$ . Draw lines through  $S = 0$  and points on the  $d$  scale. Somewhere on these lines will be found the corresponding values of  $D$ .

6. Now let  $d = 0$ ; then  $DS = 0.0982D^4$ . From this equation, we can determine values of  $S$  for given values of  $D$ .

$D$	5	6	7	8	9	10
$S$	12.3	21.2	33.7	50.3	71.6	98.2

Now draw lines through  $d = 0$ , and the values of  $S$  shown in the table above.

## Chapter Thirteen

### THE USE OF DETERMINANTS IN THE DESIGN AND CONSTRUCTION OF ALIGNMENT CHARTS

Most students find the geometric method discussed in the previous chapters a simple and direct approach to the design and construction of alignment charts. It is felt, however, that an introduction to the method which employs determinants is desirable so that students will be enabled

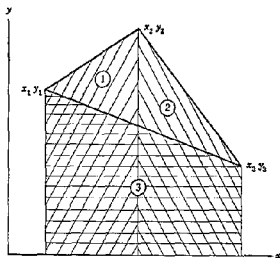


FIG. 85.

to comprehend, more fully, treatments based on determinants exclusively.

Let us consider Figure 85. The area of the triangle may be obtained by the difference between the area of trapezoid 3 and the sum of the areas of trapezoids 1 and 2; or

Area of triangle

$$= \frac{1}{2}[(y_2 + y_1)(x_2 - x_1) + (y_2 + y_3)(x_3 - x_2) - (y_1 + y_3)(x_3 - x_1)]$$

In determinant form this would be

$$A = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

If point  $x_2y_2$  were placed on the line joining  $x_1y_1$  with  $x_3y_3$ , the points are said to be co-linear, or the area of the triangle is zero. Hence

$$A = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

or when

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

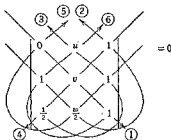
the points  $x_1y_1$ ,  $x_2y_2$ , and  $x_3y_3$  are on the same line.

### EXAMPLE 1

Suppose we have the determinant

$$\begin{vmatrix} 0 & u & 1 \\ 1 & v & 1 \\ \frac{1}{2} & \frac{w}{2} & 1 \end{vmatrix} = 0$$

1. What does the determinant mean?
  2. How can we construct an alignment chart from this determinant?
- In order to answer the first question, we must learn how to expand, or evaluate, the determinant, which is done in the following manner:



- (a) Multiply 0,  $v$ , and 1. This is step 1 (see arrow).
- (b) Multiply 1,  $w/2$ , and 1. This is step 2 (see arrow).

(c) Multiply  $\frac{1}{2}$ , 1, and  $u$ . This is step 3 (see arrow).

(d) Add the results of steps (a), (b), and (c). Thus far we have:

$$\left[0 + \frac{w}{2} + \frac{u}{2}\right]$$

Now start in the upper right-hand corner.

(e) Multiply 1,  $v$ , and  $\frac{1}{2}$ . This is step 4 (see arrow).

(f) Multiply 1,  $w/2$ , and 0. This is step 5 (see arrow).

(g) Multiply 1, 1, and  $u$ . This is step 6 (see arrow).

(h) Add the results of steps e, f, and g. This is

$$\left[\frac{v}{2} + 0 + u\right]$$

Finally, subtract  $h$  from  $d$ , i.e.,

$$\frac{w}{2} + \frac{u}{2} - \frac{v}{2} - u = 0$$

or 
$$u + v - w = 0$$

Now with regard to the second question. If we consider 0,  $u$  as  $x_1/y_1$ ; 1,  $v$  as  $x_2/y_2$ ; and  $\frac{1}{2}$ ,  $w/2$  as  $x_3/y_3$ , we may plot points for  $u$ ,  $v$ , and  $w$  by assigning definite values such as 0, 1, 2, 3  $\cdots n$  to each.

Since the  $x$  value is zero for all values of  $u$ , all points of  $u$  will lie on the  $Y$  axis. Likewise, since  $x_2 = 1$ , all points of  $v$  will lie on a line parallel to the  $Y$  axis and one unit to the right. Similarly,  $x_3 = \frac{1}{2}$ , and all points of  $w$  will lie on a line parallel to the  $Y$  axis and a  $\frac{1}{2}$  unit to the right. It should be observed that since  $y_3 = w/2$ , the distance between consecutive values of  $w$  will be half the distance between consecutive values of  $u$  or  $v$ .

A straight line which joins a point on the  $u$  scale with one on the  $v$  scale will cut the  $w$  scale in a value which satisfies the equation  $u + v = w$  (Figure 86). While the above material is easy to follow, one may wonder how the determinant

$$\begin{vmatrix} 0 & u & 1 \\ 1 & v & 1 \\ \frac{1}{2} & \frac{w}{2} & 1 \end{vmatrix} = 0$$

was developed in the first place. This could have been done by trial



and error, observing that the right-hand column must consist of ones, and that only one variable should appear in each row.

A better approach, one which is direct and mathematically correct, is this:

(a) First, write the equation  $u + v - w = 0$ .

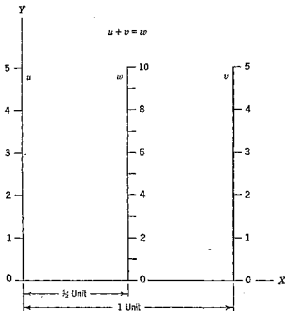


FIG. 86. Chart for the Equation,  $u + v = w$ , Constructed by the Method of Determinants.

(b) Second, let  $x = u$ , and  $y = v$ .

(c) Third, write the expressions

$$x - u = 0$$

$$y - v = 0$$

$$x + y - w = 0$$

It should be noted that we now have three equations in  $x$  and  $y$ . If they are consistent, the determinant made up from the coefficients of

$x$  and  $y$  and the constant term must vanish. [See Bocher's text *Introduction to Higher Algebra* (Chapter Four).] This means that

$$\begin{vmatrix} 1 & 0 & -u \\ 0 & 1 & -v \\ 1 & 1 & -w \end{vmatrix} = 0$$

The value of this determinant is  $u + v = w$ . You will recall that the determinant must be in the form

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

before the chart can be constructed. How can we manipulate the determinant in order to transform it to the form,

$$\begin{vmatrix} 0 & u & 1 \\ 1 & v & 1 \\ \frac{1}{2} & \frac{w}{2} & 1 \end{vmatrix} = 0$$

Let us start with

$$\begin{vmatrix} 1 & 0 & -u \\ 0 & 1 & -v \\ 1 & 1 & -w \end{vmatrix} = 0$$

Column 1 may be replaced by the sum of columns 1 and 2, yielding

$$\begin{vmatrix} 1 & 0 & -u \\ 1 & 1 & -v \\ 2 & 1 & -w \end{vmatrix} = 0 \quad (\text{Note column 3 above was multiplied by } -1.)$$

Now the bottom row may be divided by 2, resulting in

$$\begin{vmatrix} 1 & 0 & -u \\ 1 & 1 & -v \\ 1 & \frac{1}{2} & \frac{-w}{2} \end{vmatrix} = 0$$

By interchanging columns we get,

$$\begin{vmatrix} 0 & u & 1 \\ 1 & v & 1 \\ \frac{1}{2} & \frac{w}{2} & 1 \end{vmatrix} = 0$$

which is known as the "design determinant." All the steps shown above are permissible when the value of the determinant is zero. Rules for operating determinants are available in any good algebra textbook.

### EXAMPLE 2

*Given:*  $u + vw = w^2$

*Required:* The design determinant.

*Solution:* Let  $x = u$  and  $y = v$ .

$$\text{Now} \quad x - u = 0 \quad (1)$$

$$y - v = 0 \quad (2)$$

$$x + yw - w^2 = 0 \quad (3)$$

If these equations are consistent, then

$$\begin{vmatrix} 1 & 0 & -u \\ 0 & 1 & -v \\ 1 & w & -w^2 \end{vmatrix} = 0$$

Replace column one by the sum of the first two columns.

$$\begin{vmatrix} 1 & 0 & u \\ 1 & 1 & v \\ w + 1 & w & w^2 \end{vmatrix} = 0$$

Divide the bottom row by  $w + 1$ .

$$\begin{vmatrix} 1 & 0 & u \\ 1 & 1 & v \\ 1 & \frac{w}{w+1} & \frac{w^2}{w+1} \end{vmatrix} = 0$$

Rearrange the columns

$$\begin{vmatrix} 0 & u & 1 \\ 1 & v & 1 \\ \frac{w}{w+1} & \frac{w^2}{w+1} & 1 \end{vmatrix} = 0$$

Construct the chart from the above determinant (Figure 87).

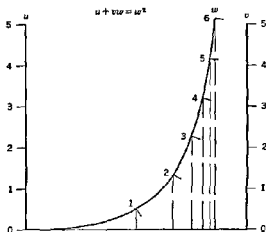


FIG. 87. Chart for the Equation,  $u + vw = w^2$ , Constructed by the Method of Determinants.

Points on the  $w$  scale can be plotted from the following co-ordinates:

	$w$	0	1	2	3	4	5
$x$ -co-ordinates:	$\frac{w}{w+1}$	0	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{6}$
$y$ -co-ordinates:	$\frac{w^2}{w+1}$	0	$\frac{1}{2}$	$\frac{4}{3}$	$\frac{9}{4}$	$\frac{16}{5}$	$\frac{25}{6}$

Up to this point no mention has been made of scale moduli. In practical alignment charts this is a significant consideration.

## EXAMPLE 3

Suppose we consider the equation  $u + v = w$  again. Let us assume that  $u$  varies from 2 to 10 and that  $v$  varies from 5 to 15. You will recall that we had written the expressions,

$$x - u = 0$$

$$y - v = 0$$

$$x + y - w = 0$$

Now, however, let us introduce the scale moduli by writing

$$x - m_u u = 0$$

$$y - m_v v = 0$$

and 
$$\frac{x}{m_u} + \frac{y}{m_v} - w = 0 \text{ (since } u + v - w = 0 \text{)}$$

We can write the determinant,

$$\begin{vmatrix} 1 & 0 & -m_u u \\ 0 & 1 & -m_v v \\ \frac{1}{m_u} & \frac{1}{m_v} & -w \end{vmatrix} = 0$$

because the three equations above are consistent. This determinant may be reduced to the "design determinant" in the following manner:

$$\begin{vmatrix} 1 & 0 & m_u u \\ 0 & 1 & m_v v \\ \frac{1}{m_u} & \frac{1}{m_v} & w \end{vmatrix} = \begin{vmatrix} 1 & 0 & m_u u \\ 1 & 1 & m_v v \\ \frac{m_u + m_v}{m_u m_v} & 1 & w \end{vmatrix} = \begin{vmatrix} 1 & 0 & m_u u \\ 1 & 1 & m_v v \\ 1 & \frac{m_u}{m_u + m_v} & \frac{m_u m_v}{m_u + m_v} w \end{vmatrix} =$$

$$\begin{vmatrix} 0 & m_u u & 1 \\ 1 & m_v v & 1 \\ \frac{m_u}{m_u + m_v} & \frac{m_u m_v}{m_u + m_v} & w \end{vmatrix} \leftarrow \text{Design Determinant}$$

If the lengths of the  $u$  and  $v$  scales are 6 in., then

$$m_u = \frac{6}{10 - 2} = \frac{3}{4}$$

and

$$m_v = \frac{6}{15 - 5} = \frac{3}{5}$$

The design determinant becomes:

$$\begin{vmatrix} 0 & \frac{3}{2}u & 1 \\ 1 & \frac{3}{8}v & 1 \\ \frac{5}{9} & \frac{1}{9}w & 1 \end{vmatrix} = 0$$

The chart is constructed from this determinant (Figure 88). It should be noted that the  $u$  and  $v$  scales are graduated from points 2

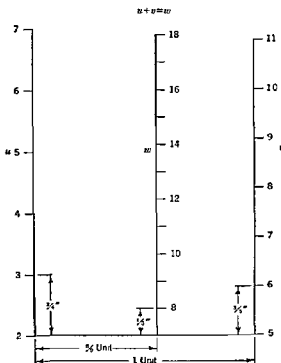


FIG. 88. Chart for the Equation,  $u + v = w$ , Constructed by the Method of Determinants.

and 5 respectively. A point on the  $w$  scale is obtained from the relation  $u + v = w$ .

## GENERAL REMARKS

1. An equation which can be reduced to the determinant,

$$\begin{vmatrix} 0 & f_1(u) & 1 \\ f_2(v) & f_3(v) & 1 \\ f_4(w) & f_5(w) & 1 \end{vmatrix} = 0$$

will consist of a straight line  $u$  scale; and curved scales for  $v$  and  $w$ .

2. If the determinant is of the form:

$$\begin{vmatrix} f_1(u) & f_2(u) & 1 \\ f_3(v) & f_4(v) & 1 \\ f_5(w) & f_6(w) & 1 \end{vmatrix} = 0$$

the alignment chart will consist of three curved scales.

3. Design determinants of the form:

$$\begin{vmatrix} 0 & f_1(u) & 1 \\ 1 & f_2(v) & 1 \\ f_3(w) & 0 & 1 \end{vmatrix} = 0$$

will result in a chart having two parallel scales ( $u$  and  $v$ ) and a transverse line for scale  $w$ .

The reduction of a given equation to the design determinant form frequently requires ingenuity and resourcefulness on the part of the designer. As one develops facility in manipulating determinants he will evolve short-cuts that will save much time.

In most cases the geometric method will be adequate for the design of alignment charts. However, complicated expressions, especially those which result in charts having two or three curves, may be solved more easily if the equation can be expressed in determinant form directly.

Projective transformations can be handled very nicely if the determinant forms are employed. The interested student is encouraged to consult the bibliography for a selection of books which stress the method of the determinants.

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## APPENDIX

These are examples of alignment charts which may prove useful in the fields of engineering, production, business, and statistics.

$$E = \frac{336 \times R \times S}{D}$$

EXAMPLE :

R = 6:1

READ ON "E"

S = 35 M.P.H.

E = 2100 R.P.M.

D = 34 IN

$$E = \frac{336 \times 6 \times 35}{34} = 2080$$

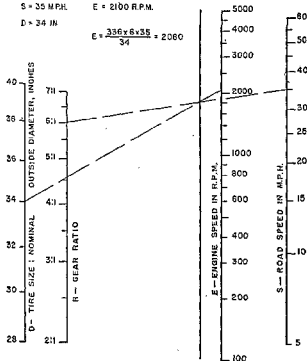


FIG. 1A. Alignment Chart to Determine Engine R.P.M.

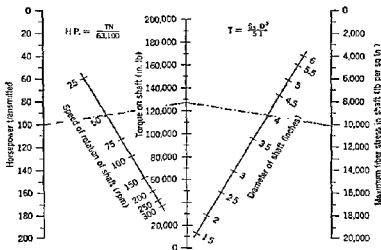


FIG. 2A. Shafts for Torsional Strength. (Courtesy *Product Engineering*.)

For calculating the horsepower that can be transmitted by a given shaft at a given speed, the following equation can be used.

$$HP = \frac{S_s D^3 N}{322,000} \quad (1)$$

This equation may be divided into two parts.

$$S_s = \frac{16T}{\pi D^3} \quad (2)$$

and

$$HP = \frac{TN}{63,100} \quad (3)$$

$S_s$  = max shearing stress,

$D$  = diameter,

$N$  = rpm,

and

$T$  = torque

Manipulation of the chart is illustrated by the dash lines. Thus, if 100 hp is to be transmitted at a speed of 50 rpm, the stress in the outer fibres of a 4-in. diam. solid shaft will be slightly in excess of 10,000 lb per sq in.

Strength of standard fine-thread bolts  
 $P = A \times S$

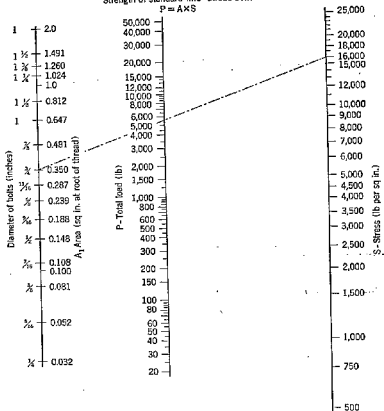


FIG. 3A. Strength of American Standard Bolts. (Courtesy *Product Engineering*.)

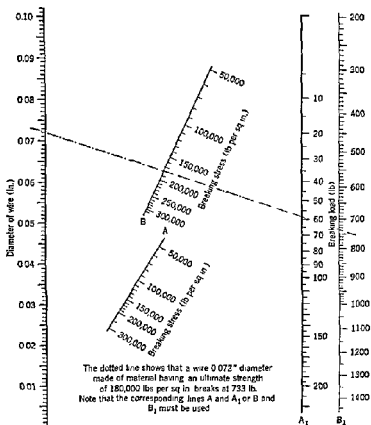


FIG. 4A. Breaking Load of Round Wires. (Courtesy *Product Engineering*)

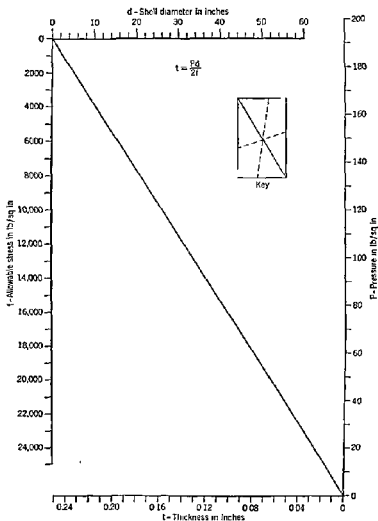


FIG. 6A. Required Shell Thickness for Various Fluid Pressures

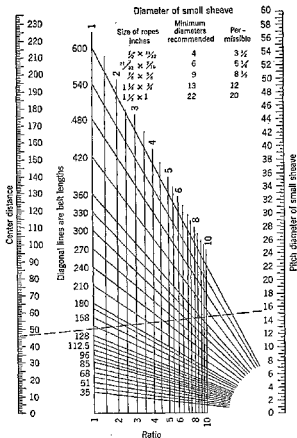


FIG. 7A. Chart for Finding Belt Lengths, V-Belt Short-Center Drives. (Courtesy Product Engineering. Information courtesy of Allis-Chalmers Manufacturing Co.)

Given pitch diameter of the small sheave, center distance, and speed ratio: to find length of V-belt, place straightedge on the given points of small sheave diameter (right) and center distance (left) and note intersection with ratio line (middle). Interpolate between diagonal belt length lines to obtain desired length.

**EXAMPLE:** Dotted line drawn between points representing small sheave diameter of 15.05 in. and center distance 46 in. intersects ratio line 2.6 at diagonal line for 180 in. belt length.

The V-belt drive consists of a driving and driven sheave, grooved for a multiplicity of belts of trapezoidal cross-section. Power is transmitted by the wedging contact between the belts and grooves. At maximum load, repeated tests show an efficiency of 99 per cent and a coefficient of friction of 1.6. V-belt drives operate, therefore, with comparatively small tension on the slack side, without slippage and with little creep. In figuring loads, it is usually safe to take 1.5 times the torque to get the total belt pull. Manufacturer's ratings must be consulted for selection of number and size of belts for given load conditions.

A V-belt drive will usually be well proportioned when the center distance equals or is slightly greater than the large sheave diameter. On small ratios the sheaves may be operated so closely together that the sheaves almost touch each other. Maximum center distance on  $\frac{1}{2}$ -in. belts is 17 in., except on high ratios, where 25 in. is permissible.

In the accompanying chart, the sheave diameters are the pitch diameters, measured at the midpoint of the trapezoidal section of the belt when resting in the groove.



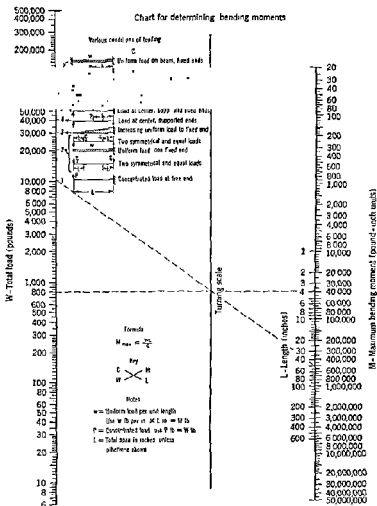


FIG. 8A. Chart for Determining Bending Moments (Courtesy Product Engineering.)

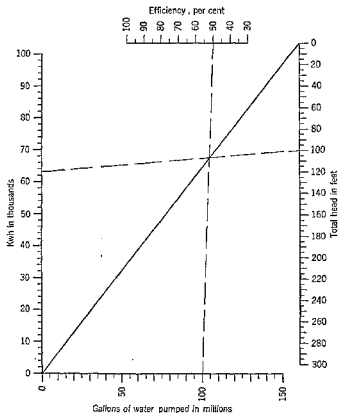


FIG. 9A. Power Required to Pump Water.

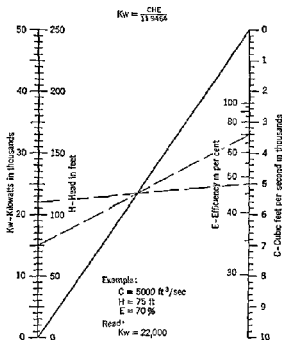


FIG. 10A. Power Developed from Stream Flow.

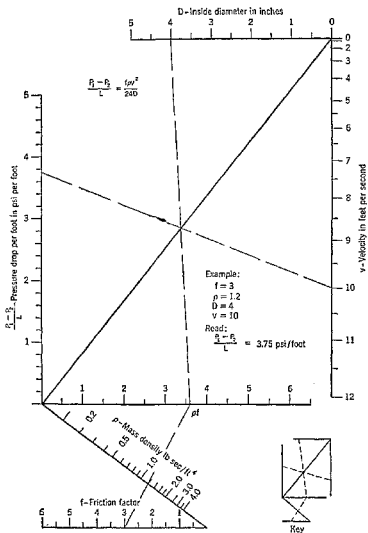


Fig. 11A. Pressure Drop in Pipe Line.

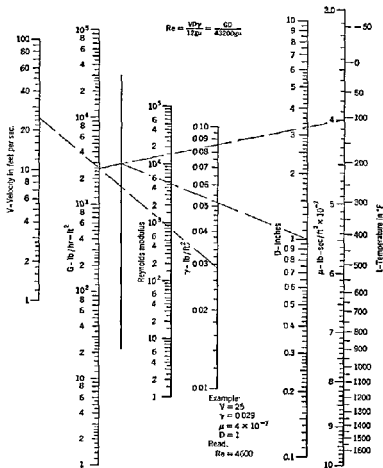


FIG. 12A. Reynolds Modulus for Air.

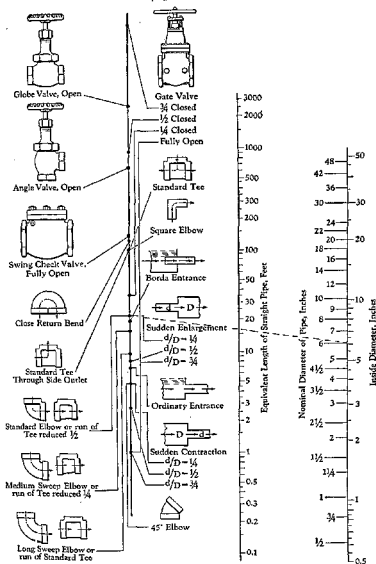


FIG. 13A Resistance of Valves and Fittings to Flow of Fluids. (Courtesy Crane Co.)

The chart on the page below solves the formula:

$$R_s = \frac{50.7Q\rho}{d\mu} = \frac{35.5B\rho}{d\mu} = \frac{6.32W}{d\mu}$$



Example: Oil is flowing through a 2-inch Standard new steel pipe at the rate of 600 barrels per hour. If the specific gravity of the oil is 0.97 and the viscosity at the flowing temperature is 14 centipoise, what is the Reynolds number and the friction factor?

FIG. 14A. Reynolds Numbers and Friction Factor for the Flow of Liquids in Pipes. (Courtesy Crane Co.)  
(Determined from the Rate of Flow, Viscosity, Pipe Diameter, and the Density or Specific Gravity.)



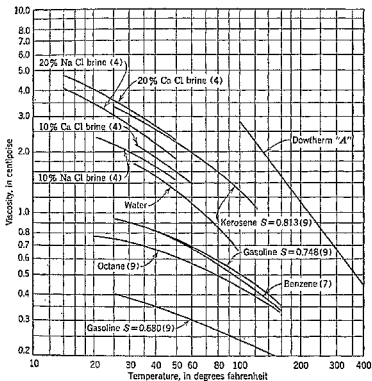


FIG. 15A. Viscosity of Various Liquids. (Courtesy Crane Co.)



It is good practise to assume that flow in pipes is turbulent for all Reynolds numbers greater than 1200. The chart on the page below is a solution of the Fanning formula for turbulent flow.

$$\Delta P_{\text{res}} = 0.129 \frac{f \rho v^3}{d} = 0.0215 \frac{f \rho Q^2}{d^5} = 0.0106 \frac{f \rho B^3}{d^5}$$

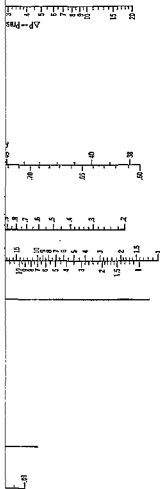


FIG. 16A. Pressure Drop in Liquid Lines (Turbulent Flow). (Courtesy Crane Co.)  
(Determined from Rate of Flow, Friction Factor, Pipe Diameter, and Density.)

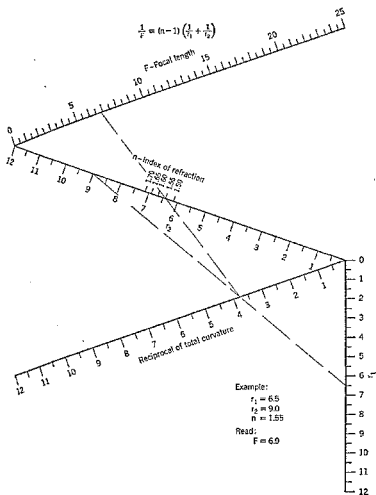


FIG. 18A. Focal Length of Thin Lenses.

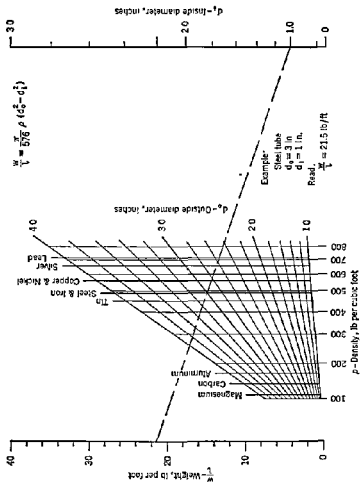
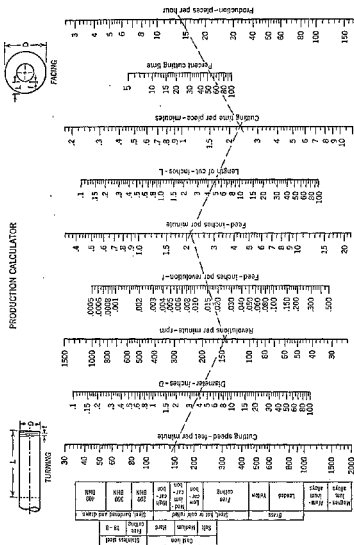


Fig. 10A Weight of Tubing.



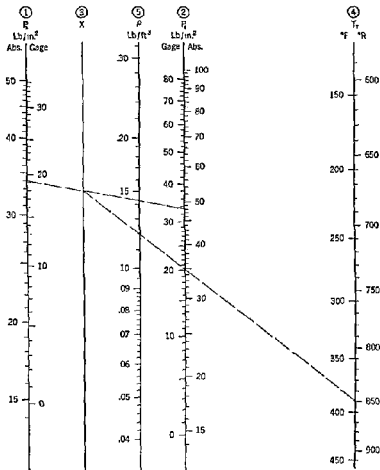


FIG. 21A. Nomogram for Solving for Density of Fluid in a Pitot Tube Traverse.  
(Courtesy Consolidated Vultee Aircraft Corp.)

$$\rho = \frac{P_s}{R} \times \frac{1}{T_r} \times \left( \frac{P_T}{P_s} \right)^{\frac{K-1}{K}}, \text{ lb/ft}^3$$

$$K = 1.4 \text{ (for air)}$$

where  $R = 53.3 \text{ ft.-lb./}^\circ\text{F./lb. of air}$

$T_r$  = stagnation temp.,  $^\circ\text{R}$

$P_T$  = stagnation pres. } similar units

$P_s$  = static pres. }

*Example* Given:  $P_s = 34.0 \text{ lb/in}^2 \text{ abs.}$ ,  $P_T = 48.0 \text{ lb/in}^2 \text{ abs.}$ ,  $T_r = 850^\circ\text{R}$

Connect 34.0 on scale 1 to 48.0 on scale 2 and thus locate point X on scale 3. Connect point X with 850 on scale 4 and read answer,  $\rho = 0.1195$ , on scale 5.

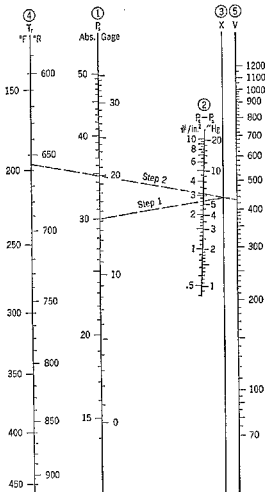


FIG. 22A. Nomogram for Solving the Pitot Tube Formula for Compressible Fluids.  
(Courtesy Consolidated Vultee Aircraft Corp.)

$$V = \sqrt{2gJ C_p T_T \left[ 1 - \left( \frac{P_s}{P_T} \right)^{\frac{K-1}{K}} \right]} \text{ ft./sec}$$

where  $g = 32.17 \text{ ft./sec}^2$ ,  $J = 778 \text{ ft.-lb./BTU}$ ,  $K = 1.4$ ,  $C_p = 0.24 \text{ BTU/°F/lb air}$

$T_T$  = stagnation temp., °R

$P_T$  = stagnation pres., lb/in.² abs. or similar units

$P_s$  = static pres., lb/in.² abs. or similar units

Example. Given:  $P_s = 30.0 \text{ lb/in.}^2 \text{ abs.}$ ,  $P_T - P_s = 2.0 \text{ lb/in.}^2$ ,  $T_T = 100^\circ\text{F}$ .

Connect 30.0 on scale 1 to 2.0 on scale 2 and note point X on scale 3. Connect this point with 100 on scale 4 and read answer on scale 5,  $V = 420 \text{ ft./sec}$ .

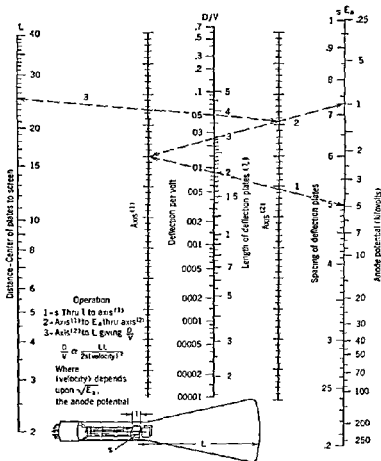


FIG. 23A. Deflection Sensitivity of Cathode Ray Tube—Electrostatic Deflection.  
 (Courtesy Federal Telephone and Radio Corp.)

The electrostatic deflection sensitivity of a cathode ray tube, for a given anode potential ( $E_a$ ), depends on spacing ( $s$ ) and length ( $l$ ) of deflection plates, and their distance ( $L$ ) to the screen. A line from ( $l$ ) and ( $s$ ) scales is extended to axis (1). From latter intersection a second line is drawn to ( $E_a$ ) scale intersecting axis (2). From this point, a third line is drawn through ( $L$ ) and extended to intersect  $D/V$  scale at left. ( $s$ ), ( $l$ ), ( $L$ ), and ( $D$ ) may all be in any convenient system of dimensions.

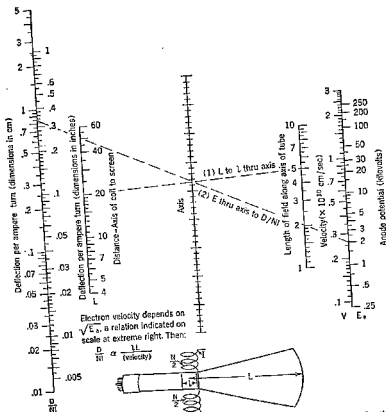


FIG. 24A. Deflection Sensitivity of Cathode Ray Tube—Electromagnetic Deflection.  
(Courtesy Federal Telephone and Radio Corp.)

The electromagnetic deflection sensitivity of a cathode ray tube cannot be determined with accuracy owing to difficulty of computing intensity and dimensions of magnetic field. With no flux leakage  $H = .04\pi NI$ , and the approximate deflection can be determined from chart, as indicated. The deflection per ampere turn scales (for use when dimensional systems are in centimeters and in inches) are at left. The scale at right shows electron velocity resulting from a given anode potential.



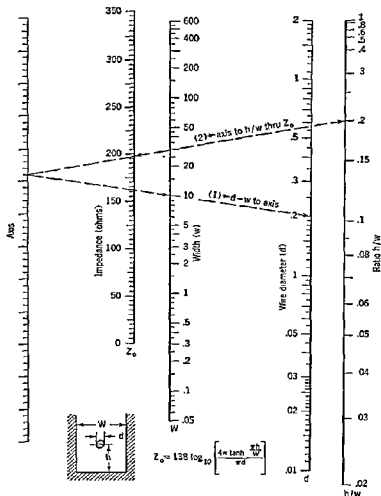


FIG. 25A. Characteristic Impedance of Lines—Single Wire in Trough. (Courtesy Federal Telephone and Radio Corp.)

Start with  $d$  and  $W$  values and extend line to axis. From latter point, a line to  $h/w$  scale will intersect  $Z_0$  axis at resulting impedance value. Engineering accuracy when  $w/d > 5$  and  $h/d > 1.5$ . Relations are only approximate beyond these values. Assumes lossless dielectric (air) and perfect conductors. Sides of trough assumed to extend to infinity. Any dimensional unit may be used. For other dielectrics multiply  $Z_0$  by  $1/\sqrt{\epsilon}$  where  $\epsilon$  is dielectric constant.

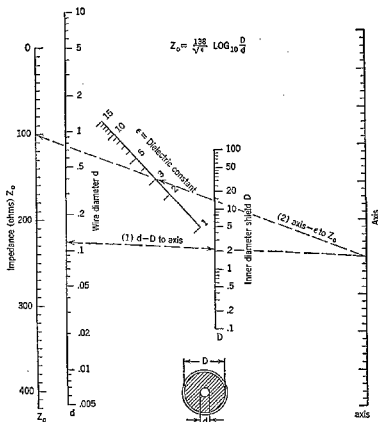


FIG. 28A. Characteristic Impedance of Lines—Concentric Line, Solid Dielectric. (Courtesy Federal Telephone and Radio Corp.)

This chart gives theoretically exact values for any scale of dimensions, if lossless dielectric is assumed (filling completely the space between conductors), and perfect conductors. Line from  $d$  and  $D$  scales extends to the axis. From latter point a line through  $E$  scale will intersect  $Z_0$  scale at resulting value.

Example:  $d = .12$  cm,  $D = 2.1$  cm,  $E = 3.0$ ,  $Z_0 = 102$  ohms.

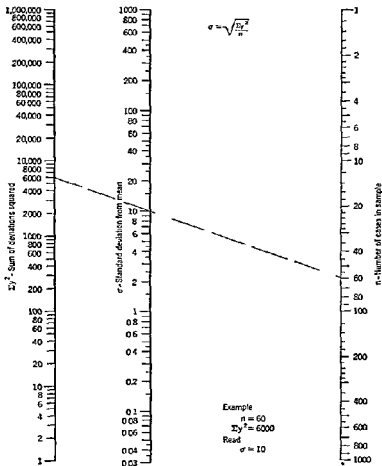


FIG. 27A. Standard Deviation of a Set of Scores.

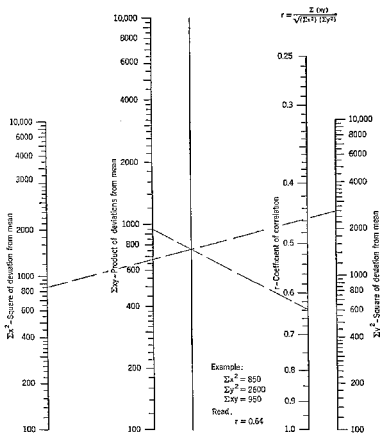


FIG. 28A. Coefficient of Correlation.

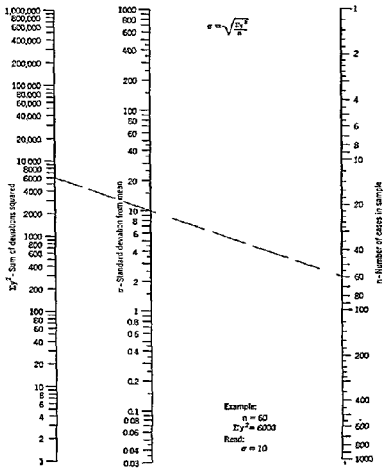


FIG. 27A. Standard Deviation of a Set of Scores.

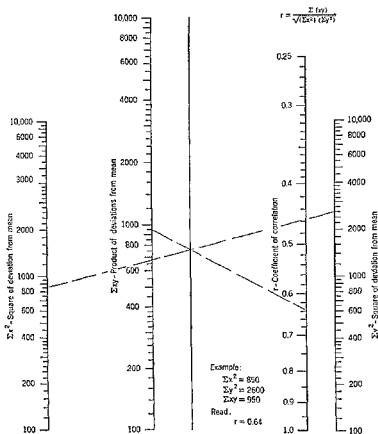


FIG. 28A. Coefficient of Correlation.

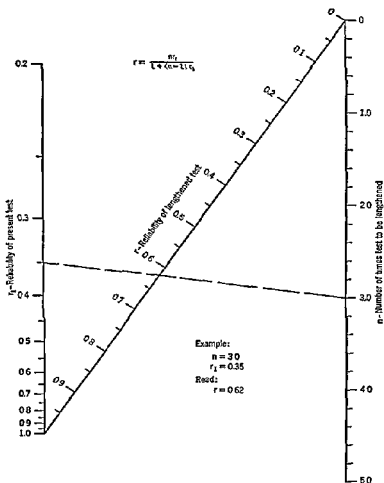


FIG. 29A. Spearman-Brown Prophecy Formula for Reliability of Lengthened Test.

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